Charitable giving: Altruism has no limits

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Abstract

The current consensus is that, due to the free rider problem, a theory of charitable giving based exclusively on altruism cannot explain the levels of contributions and participation observed in the data. In contrast, in this paper we demonstrate that altruism may have more predictive power than previously understood. We present a model in which the object of altruism is a large number of potential recipients of a good for which there is a target level of provision (e.g. an acute malnutrition treatment, an insecticide-treated bed net). We show that in this case donors may measure the value of a contribution with reference to the impact that such contribution has on additional recipients, not with reference to the impact on the total level of an organization's charitable resources as traditionally assumed. In this situation, the equilibrium involves no free riding whatsoever. Equilibria that do involve free riding are characterized by provision of the target level of the charitable good to all potential recipients.

KEYWORDS: Altruism, free-riding, indivisible charitable goods.

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We believe in a world where ZERO children die of things we can prevent.
UNICEF (Believe in Zero campaign)

It’s unacceptable that a child dies every minute for lack of malaria diagnosis and treatment, worth only a dollar. Malaria No More (Power of One campaign)

1 Introduction

It has been argued that large charitable organizations like the American Red Cross, UNICEF, or Medicins Sans Frontières are a logical impossibility within a theory of private provision of public goods based exclusively on altruism (e.g. Sugden 1982, 1984, 1985; Andreoni, 1988). According to the traditional theory, for example, only the wealthiest individuals would donate to provide a public good when the number of potential donors is large enough (Andreoni, 1988). Thus, if a large private foundation (e.g. The Bill and Melinda Gates Foundation) were to participate in the provision of a public good, almost everyone else would free ride. Yet, as Rose-Ackerman (1996 p.712) points out, "in contradiction to strong models of free rider behavior, many people donate both time and money to organizations where their own gifts have only an insignificant impact on the level of services provided." Furthermore, contrary to observed behavior, the traditional theory implies that government contributions funded by lump sum taxes, or even exogenous increases in contributions, will have at best a negligible effect in the equilibrium levels of contributions for charitable organizations with a broad donors’ base. That is, government provision will completely -or almost completely- crowd out private provision (e.g. Warr, 1982; Roberts, 1984; Bergstrom et al., 1986; Andreoni, 1988).

Given these counterfactual predictions, Andreoni (1988 pg.57) concludes that "A truly descriptive model of privately provided public goods must be generalized to include other non-altruistic motives for giving." For example, it has been argued that people may make donations to charities because they derive satisfaction from the act of giving (Andreoni 1989, 1990) or from the feeling of ‘making a difference’ (Duncan, 2004), to receive acclaim or prestige in society, (Glazer and Konrad, 1996; Harbaugh 1998a,b), or because they are constrained by moral principles (e.g. Sugden, 1984) -See Andreoni (2006) for a review of the literature-. While we believe that these are all important motives for giving, an alternative explanation of the observed regularities could be that the classical model is misspecified. That is, instead of additional motives for giving, it may be that the main motive for charitable giving is indeed altruism, but that the traditional model does not
capture important peculiarities of charitable giving. Our objective in this paper is to present a theory that formalizes an alternative explanation of philanthropic activity based exclusively on altruistic behavior.

Our departure from existing theories is based on two simple observations. The first observation is that charitable organizations that provide basic necessities to disadvantaged individuals and that have a large donors' base also have a large number of potential recipients. This fact has been neglected in existing theories, where the nature of the public good under consideration is left unspecified. Whether the public good characterizes meals provided by a local charity or vaccines provided by UNICEF to more than a third of the world's children is not entirely clear. In effect, under the existing theories it is not obvious why the equilibrium level of contributions to UNICEF would be greater than that to a given local charity. We argue and formally demonstrate that this observation is of critical importance for understanding charitable giving since the extent of free riding is likely determined by the number of potential donors relative to the number of potential recipients.

The second observation is that existing theories of charitable giving can generate gifts that are "too small" to have any effectiveness on any particular recipient. To exemplify what we mean, suppose that the charitable good refers to insecticide-treated bed nets. Suppose that, in a given time period, there are one million potential recipients, that total contributions to the charitable organization is one million dollars, and that the cost of the nets is five dollars per unit. In this case, any model that assumes convex consumption sets would conclude that each recipient would consume 0.2 units of bed nets. This is an absurd implication. Of course, this would not be a significant problem for the existing theories if non-convexities were exceptional in charitable giving. However, since the reason for the existence of many charitable organizations is providing basic necessities to disadvantaged individuals, non-convexities in the form of subsistence levels of consumption or in the form of indivisible goods and services are the norm rather than the exception.

Based on these observations, we present a model in which the object of altruism is a large number of potential recipients of a good or service for which there is a minimum efficient level of provision (e.g. a nutrition package, a vaccine, a diagnostic test, a cancer or HIV treatment, a bed net, a doctor or a tutor's time). We show that in this case potential donors may measure the value of a contribution with reference to the impact that such contribution has on additional individual recipients, not with reference to the impact that their contribution has on the total level of an organization's charitable resources.
or the impact it has on a given charitable cause as assumed in the existing theories. Therefore, for example, when a donor decides to give 50 additional dollars to UNICEF, the donor may be well aware that the impact of this donation is insignificant on UNICEF’s total level of funding. Furthermore, the donor may be aware that the impact of this donation on the charitable cause (e.g. to eliminate global hunger) is insignificant. However, the purpose and consequence of the donor’s gift may be to provide a therapeutic-food treatment to an additional malnourished child that, without her donation, would likely die. As long as there exist individuals that are severely malnourished and or dying from preventable or curable diseases, each additional contribution could have a large impact, irrespective of the fact that The Bill and Melinda Gates Foundation and millions of other donors may contribute to the same organization or, more generally, to the same charitable cause.\footnote{Importantly, contrary to Duncan’s (2004) model of impact philanthropy, in our model it is not just the feeling of making a difference that drives charitable giving. Instead, in our model even small contributions can actually have a large impact on the lives of additional recipients of charitable goods.} Indeed, the introductory quotes suggest that charitable organizations seek to convey precisely this message to potential donors.

Formally, in our model altruism is captured by a standard aggregation rule (i.e. the sum of utilities of potential recipients of a charitable good), so donors view the utilities of the potential recipients as perfect substitutes but their consumption levels as imperfect substitutes. Given a target level of provision per recipient and a large number of potential recipients, there exists an equilibrium in which additional contributions change the number of recipients of the charitable good and each donor contributes the amount that is individually optimal independently of the amount contributed by other donors. In this equilibrium, a lump sum tax on non-contributors, a tax in excess of the equilibrium contributions, or redistribution of income from non-contributors to contributors (more generally, any exogenous increase in the charity’s budget), may increase the total level of contributions dollar-for-dollar.

While there may be equilibria that do involve crowding out, all of these equilibria are characterized by the efficient outcome in which the target level is provided to all potential recipients. This will occur whenever the sum of the amounts that each donor is willing to contribute independently of the contributions of others exceeds the cost of providing the target level of the charitable good to all recipients. This is similar in spirit to the findings in the literature on private provision of discrete public goods (e.g. Palfrey and Rosenthal, 1984; Bagnoli and Lipman 1989, 1992; Admati and Perry, 1991) where it has been shown
that, when there is a single unit of a public good to be supplied, the outcome may be efficient (i.e. the public good is provided whenever the sum of each donor’s valuation of the good is greater than the cost of provision).²

The rest of the paper formalizes these arguments. The next section revisits the implications of the classic model of private provision of public goods. In Section 3, we look closer at the notion of altruistic preferences and we discuss how (and when) the traditional framework captures altruism. In Section 4 we extend the basic framework to analyze indivisibilities in the provision of charitable goods. Section 5 shows how the results can be applied to understand some of the empirical regularities about charitable giving in the context of disaster relief efforts. Section 6 provides concluding remarks.

2 The standard model

Let us recast the standard model of voluntary provision of public goods as applied to charitable activities. Suppose that there are \( N \) potential contributors that consume two goods. One is a private commodity. The other one is a non-excludable and non-rival good that is provided by a charitable organization through the joint funding by the contributors. Each contributor \( i \) \( (i = 1, \ldots, N) \) has an endowment of income \( Y_i \) and contributes \( g_i \) towards the provision of the public good. In addition, it is assumed that each contributor has a convex consumption possibilities set, a smooth, strictly increasing in each argument, and quasiconcave bivariate utility function of the form \( U_i(c_i, G) \), and that the level of the public good produced and consumed by all donors equals the sum of contributions \( G = \sum_{i=1}^{N} g_i \). Taking as given the contributions of other donors, \( G_{-i} \), donor \( i \) \( (i = 1, \ldots, N) \) solves the following program

\[
\begin{align*}
\max_{c_i, g_i} & \quad U_i(c_i, G) \\
\text{s.t.} & \quad Y_i = c_i + g_i \\
& \quad 0 \leq g_i \leq Y_i \\
& \quad 0 \leq c_i \leq Y_i \\
& \quad G = g_i + G_{-i}
\end{align*}
\]

The following implications of the model are well known by now.

²There are other links between our analysis and this literature that we discuss in more depth in Section 4.
Result 1. **Level of provision.** For $N$ large, average giving is close to zero (Andreoni, 1988).

Result 2. **Level of participation.** For $N$ large, the proportion of donors contributing to the public good is close to zero; only the richest donors contribute (Andreoni, 1988).

Result 3. **Crowding out.** Government contributions to the public good, financed by lump sum taxes, decrease private contributions dollar-for-dollar as long as the taxes do not exceed the private contributions (Warr, 1982; Roberts, 1984; Bergstrom et al. 1986). For $N$ large, a tax in excess of private contributions increases equilibrium contributions by an imperceptible small amount (Andreoni, 1988).

Result 4. **Redistribution of income.** The level of the public good provided is neutral to redistribution of income among contributors or among non-contributors (Warr, 1982; Bergstrom et al., 1986). For large $N$, redistribution of income from non-contributors to contributors increases total contributions by an imperceptible small amount (Andreoni, 1988).

Result 5. **Mana from Heaven.** For large $N$, an exogenous increase in donations (e.g. additional contributors) increases the equilibrium level of contributions by an imperceptible small amount (Sugden, 1982; Andreoni, 1988).

All of these results run counter to the observed empirical regularities of fairly high levels of provision, broadbase participation in the private provision of public goods, and incomplete crowding out. The inference drawn from these results is that organizations like UNICEF or Medicins Sans Frontieres, where millions of "small" individual donors contribute together with very wealthy individual donors, corporations, and governments from several countries are a logical impossibility within the traditional analysis. In the next section we take a closer look at this seemingly logical inference.

3 Opening the black-box of altruistic preferences

Altruism refers to the unselfish concern over the wellbeing of others. Thus, any model that seeks to incorporate altruism into individual choice must deal with individual preferences over social states. A complete treatment of this issue would require an understanding of the highly heterogeneous and complex views that individuals have about their rights and responsibilities towards other members of society (e.g. the emotional attachment to other members of society, individual views of moral responsibilities, their perceptions of distributive justice and how injustices should be dealt with). While this treatment is cer-
tainly important to understand the different motivations behind philanthropic activities, our objective in this section is more limited. In particular, we will take as given a simple, specific notion of preferences over social states and then evaluate how the utility function $U_i(c_i, G)$ can be made consistent with this notion of altruism and whether this view is inconsistent with the observed regularities regarding philanthropic activities.

Imagine a society with $T$ individuals and suppose that there is a vector of private market commodities consumed by each individual in amount $X_i$ ($i = 1, \ldots, T$) and a vector of public commodities or services consumed in amount $Q$. Suppose that each individual derives satisfaction from consuming these goods and services and let $u_i(X_i, Q)$ denote the expected private utility from consumption. If individuals are altruistic, they may not only care about their own wellbeing but they may also have a concern for the wellbeing of others. Thus, let us consider an aggregation rule that captures the overall welfare of a given individual as follows:

$$W_i(X_1, \ldots, X_T, Q) = u_i(X_i, Q) + \sum_{j \neq i} \beta_{i,j} v_{i,j}(X_j, Q)$$

(2)

We will refer to the first term on the right hand side as private utility and to the second term (i.e. the summation) as altruistic utility. The function $v_{i,j}(X_j, Q)$ admits two possible interpretations. First, it may capture individual $j$’s private utility, so $v_{i,j}(X_j, Q) = u_j(X_j, Q)$. Second, $v_{i,j}(X_j, Q)$ may be interpreted as a function that represents the (paternalistic) preferences of the evaluator over the consumption bundles of other members of society.\(^3\) In either case, the parameter $\beta_{i,j}$ measures the degree of altruism of individual $i$ towards individual $j$. Thus, an individual is altruistic if $\beta_{i,j} > 0$ for at least some $j \neq i$ and he or she is purely egoistic if $\beta_{i,j} = 0$ for all $j \neq i$. Naturally, we expect an individual’s private utility to have greater weight in his or her social calculations, so $\beta_{i,j} < 1$.

This framework can capture two basic motivations for philanthropic activity. First, a philanthropic organization may provide a public commodity or service -i.e. some subset

\(^3\)We emphasize that (2) is an example from a wider range of possible specifications of altruism (see Johansson (1997) and Kolm (2006) for alternative representations). A more general aggregation rule, proposed by Becker (1974), is the following $W_i = u_i(X_i, Q, W_{-i})$. In this case, an individual cares not only about the felicity that other members of society derive from private consumption, but also about the other member’s feelings of altruism towards others, including oneself. Moreover, unlike our specification, in the more general case the welfare of other members of society may interact in the private utility with individual $i$’s private consumption. While we will focus on the simpler specification (2), we believe that the more general rule should have similar implications.
of Q—(e.g., National public radio, art galleries, basic medical research, environmental conservation, religious services). 

Second, due to altruism, the wellbeing of others, and by extension their private consumption, also acts as a public good, in the sense that an increase in the wellbeing of a given person is a non-excludable and non-rival good that is enjoyed by other members of society. Thus, a philanthropic organization may provide private commodities or services (e.g., vaccines, meals, clothing, medical services and treatments, disaster relief) which, by the construct of altruism, become public goods.

We can now return to the question: How can $U_i(c_i, G)$ be interpreted in terms of the aggregation rule (2)? One possible interpretation is that the problem characterizes the voluntary provision of a public commodity or service and donors are purely egoistic, so $U_i = u_i$ and $\beta_{i,j} = 0$ for all $j \neq i$. Clearly, while this problem with purely egoistic individuals may have difficulties explaining the empirical regularities, a model that incorporates altruism towards other donors does not. If individuals are altruistic, the welfare that other donors receive from the provision of the public good would enter the private calculations of how much to contribute. As a result, while the outcome may not be efficient due to some discounting of others’ utility, the level of contributions and participation needs not be small.

An alternative interpretation, consistent with an altruistic motive for giving, is that the second argument in the utility function $U_i(c_i, G)$ characterizes the consumption of a private good by a subset of disadvantaged individuals. Under this interpretation, there is a group of recipients whose private consumption is a function of the total amount contributed by other members of society to the charitable organization. For example, if we let the group of potential beneficiaries correspond to the last $T - N$ members of society, and disregarding other public commodities, we may use Equation (2) to write

$$U_i(c_i, G) = u_i(c_i) + \sum_{j=N+1}^{T} \beta_{i,j} v_{i,j}(c_j(G))$$

This is probably the representation of preferences that researchers have in mind when arguing that altruism, by itself, is incompatible with the observed empirical regularities. Let us show with a simple example that this representation may lead to very high levels

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4In principle, contributions to any charitable cause may enter this category if the subject of the charitable cause (e.g., global hunger) is seen as producing a negative externality (e.g., psychological discomfort) on potential donors and the charitable organizations as providing a service—again, a subset of Q—that ameliorates the externality. Andreoni’s (1989, 1990) warm-glow model, in which private donations also enter the set of private consumption $X_i$, seems to be most relevant in this scenario.
of provision of charitable goods even when the number of donors is large. Let \( T = N + K \). \( K \) individuals have zero resources while the other \( N \) individuals have identical resources \( Y \) and identical utility functions \( u(c_i) + \beta \sum u(c_j), \ i = 1, \ldots, N \) and \( j = 1, \ldots, K \), where it is assumed that \( u(c) \) is strictly concave. A charitable organization collects contributions and distributes them in equal shares among the set of initially disadvantaged beneficiaries, so \( c_j = \frac{g_i}{K} \). Since beneficiaries are identical post-distribution, each contributor selects \( \max_{g \geq 0} u(Y - g_i) + \beta K u \left( \frac{\sum g_i}{K} \right) \). This is a standard model of charitable giving. The only departure from the traditional framework, which makes the model much closer to reality, is that there are multiple beneficiaries of charitable gifts. This model has a unique symmetric Nash equilibrium with contributions \( g^N \) characterized by the condition \(-u'(Y - g^N) + \beta u'(\lambda g^N) = 0\), where \( \lambda = \frac{N}{K} \) is the ratio of contributors to beneficiaries. It is simple to show that \( g^N \) is increasing in the degree of altruism \( \beta \) and the number of beneficiaries \( K \). Moreover, contributions per beneficiary \( \lambda g^N (K) \) fall as the number of beneficiaries grows larger. Within a non-strategic framework, Andreoni (2007) labeled this phenomenon "congestion of altruism" and finds it to be consistent with the choices of experimental subjects.

The equilibrium level of contributions is decreasing in the number of contributors, which is the well-known free riding effect. In fact, all of the results in the previous section apply to the current framework. However, this setting also shows that establishing the limiting behavior of contributions as the number of donors grows large (e.g. Andreoni, 1988; Ribar and Wilhelm, 2003) is not too informative in the context of organizations like UNICEF or Medecins Sans Frontières that target millions of potential recipients. Suppose, for example, that \( \beta = 1 \), so contributors weight each beneficiaries' welfare as much as their own, and \( \lambda = 1 \), so there is an equal number of contributors and beneficiaries. Then, in a Nash equilibrium we have \( g^N = c_j = c_i = \frac{Y}{2} \) for all \( i \) and \( j \). That is, each individual gives half of his or her resources to charity and the Nash equilibrium level of resources is the same for all members of society. This is true whether there are ten or ten million donors (as long as we maintain \( \lambda = 1 \)).

One may argue that this particular example is based on unrealistic levels of altruism.\(^5\) Although ultimately this is an empirical matter, we do not necessarily disagree. The main significance of this example, however, is to illustrate the importance of considering the provision of charitable goods to multiple disadvantaged individuals when evaluating the

\(^5\)Specifically, with a large number of potential recipients, \( \beta = 1 \) implies that donors perceive altruistic utility to be much more important than private utility.
extent of free-riding in charitable giving. In the next section we will show that the presence of multiple recipients, together with indivisibilities in the charitable goods provided, may lead to large levels of provision and participation even for less extreme levels of altruism.

4 A model of private provision of basic necessities

There is at least one highly unsatisfactory feature of the traditional model as applied to charitable giving. Namely, these models can generate levels of contributions that, if distributed equally among all potential beneficiaries, may in fact be completely worthless to all beneficiaries, as in the example of bed nets mentioned in the introduction. Our objective in this section is to adapt the traditional framework in order to capture the indivisibilities that are ubiquitous in charitable giving.

Suppose that there is a single charitable organization, \( N \) potential donors to the charity, and a continuous measure \( K \) of potential beneficiaries of the charitable good.\(^6\) To be concrete we will frequently refer to the charitable good as a nutrition package;\(^7\) however, the framework is equally applicable to other goods or services (or baskets of different goods).

We assume that the charity uses donations to purchase nutrition packages in the market and to transport and distribute them to the donees. The dollar cost of provision per unit of the charitable good, including per unit transportation and distribution costs, is \( q \). We assume that there is a level of the charitable good \( c^\ast \) such that consumption below this level is completely ineffective. There are two possible interpretations for \( c^\ast \). First, it may represent a minimum subsistence level of consumption. For example, a grain of rice a day or a single second of a doctor’s time may be completely ineffective. Second, the charitable goods or services provided may have indivisibility features. For example, half of an insecticide-treated bed net or a fraction of an acute malnutrition treatment or diagnostic test would be of little help to potential beneficiaries. For now, we will focus

\(^6\)This last assumption may seem somewhat odd given our focus on indivisible charitable goods. However, this assumption proves useful as a benchmark to provide intuition. We look at the case of a countable set of beneficiaries in Section 4.3.1 below.

\(^7\)A specific example is UNICEF’s provision of ready-to-use therapeutic food (RUTF) for children with severe acute malnutrition. According to UNICEF (2013), around 20 million children worldwide suffer from severe acute malnutrition and about 10% receive RUTFs, a large fraction of which is provided by UNICEF. A typical treatment involves consumption of 150 sachets (1 carton) of RUTF during a 6-8 weeks period. The cost (FCA) of an RUTF carton for UNICEF in 2013 was between 45 and 75 USD depending on the vendor (http://www.unicef.org/supply/files/RUTF_Pricing_Data.pdf).
on the indivisibility aspect. Thus, we assume that the charity allocates its entire level of funding $G$ equally among all or a subset $M$ of potential beneficiaries by purchasing $c$ units of the good per person at a cost $q$ per unit: $G = q\bar{c}M$, with $M \leq K$.

On the donors side, each donor $i$ ($i = 1, \ldots, N$) with income $Y_i$ selects his private consumption $c_i$ and his monetary contribution to the charity $g_i$ taking into account how the charity allocates its available funds and taking as given the contributions of other donors. We assume that each donor has a strictly concave private utility $u_i(c_i)$, with the property $\lim_{c_i \to 0} u_i'(c_i) = \infty$, which implies that $c_i$ should be interpreted as an aggregate bundle of consumption above subsistence. In addition, since potential beneficiaries are ex-ante identical, we write the altruistic utility of donor $i$ per beneficiary as $\beta_i v_i(c)$, with $c = \{\bar{c}, 0\}$, where we assume that $v_i(c)$ is strictly concave, $v_i(\bar{c}) > v_i(0)$ and $v_i(0) < 0$ is finite.\footnote{An alternative way to interpret this model is that altruism is strictly need-based, in the sense that donors are indifferent about the wellbeing of individuals with consumption levels above the minimum subsistence level, so $\beta_i = 0$ for all $c > \bar{c}$.}

The problem for donor $i$ ($i = 1, \ldots, N$) can be depicted as follows

$$\max_{g_i, c_i} u_i(c_i) + \beta_i [M v_i(c) + (K - M) v_i(0)]$$

s.t. $Y_i = c_i + g_i$

$0 \leq g_i \leq Y_i$

$0 \leq c_i \leq Y_i$

$G = \sum_{l=1}^{N} g_l = q\bar{c}M$

$M \leq K$

Our next objective is to characterize the set of equilibria of this problem.

### 4.1 Partial provision equilibrium

Let us first conjecture the existence of an equilibrium in which the inequality $M \leq K$ is non-binding, which we will refer to as a partial provision equilibrium. In this case, the objective function can be written as follows:\footnote{Notice that the problem for a given donor in a partial provision equilibrium can equivalently be interpreted in terms of the common practice of sponsoring disadvantaged individuals. That is, we can think of donor $i$ as selecting the number of beneficiaries to sponsor, say $M_i$, subject to the constraint that the cost of sponsoring each individual equals $q\bar{c}$, so the total level of contributions for this donor equals $M_i q\bar{c} = g_i$ and the total number of sponsored beneficiaries equals $M = \sum_i M_i = \frac{G}{q\bar{c}}$.}

$$\max_{g_i} u_i(c_i) + \beta_i [M v_i(c) + (K - M) v_i(0)]$$

s.t. $Y_i = c_i + g_i$

$0 \leq g_i \leq Y_i$

$0 \leq c_i \leq Y_i$

$G = \sum_{l=1}^{N} g_l = q\bar{c}M$

$M \leq K$
The term in square brackets has a very intuitive meaning: It represents a cost-effectiveness measure of the charitable good as perceived by a given contributor. Since it plays a very important role in the analysis that follows we define this ratio formally,

**Definition 1** We say that $V_i = \frac{\beta_i(v_i(c) - v_i(0))}{q_e} + K\beta_i v_i(0)$ is the value of charitable giving (as perceived by contributor i).

For example, if the charitable good is a life saving intervention, the ratio can be interpreted as the perceived value of saving a life. More generally, in the context of health-improving charitable goods or services, we can think of $v_i(c) - v_i(0)$ as one of the standard measures of effectiveness used in the evaluation of health interventions (e.g., quality adjusted life years gained), in which case $V_i$ measures the discounted utility of health gained (e.g. discounted QALYs gained) per dollar of investment.

Provided that this equilibrium exists, an interior level of private consumption can be characterized by the first order condition

$$u_i'(c_i) = V_i.$$ (5)

That is, the contributor equalizes the marginal benefit of his private consumption $u_i'(c_i)$ to its opportunity cost, the value of charitable giving $V_i$. Using the constraint $c_i \leq Y_i$, the level of private consumption is determined as follows

$$c_i = \min (d_i(V_i), Y_i)$$ (6)

where $d_i(V_i) = u_i^{-1}(V_i)$, is the demand for private consumption, with $d_i'(V_i) < 0$ giving decreasing marginal utility.

This, in turn, uniquely determines the level of contributions

$$g_i = \max [Y_i - d_i(V_i), 0]$$ (7)

These conditions have a very intuitive interpretation. When the equilibrium level of donations is below the cost of providing the charitable good to all potential beneficiaries, the utility function of each contributor is quasilinear. As a result, contributors treat their private consumption as a necessity and charitable giving as a luxury. There is a threshold level of income $Y_i^T$ below which donor i spends all his income in his private consumption.
If, after taking into account the benefits and costs of private consumption, this contributor decides to spend $Y^T_i$ in his private consumption and decides to donate to the charitable organization, it must be because the value of saving/extend a life is greater to him than the value of extra private consumption, so he will give all his additional income to charity. Moreover, as long as there remain potential beneficiaries that lack some basic necessities that are the object of charitable giving, each additional contribution has the potential to save/extend an additional life and, as a result, a contributor behaves in the same manner that he would if he were the only contributor, i.e. there is no free-riding whatsoever among the contributors.

The income threshold $Y^T_i$ is determined by the income level that is equal to the demand for private consumption $d_i$, which is a decreasing function of the value of charitable giving. Thus, as expected, a higher perceived value of charitable giving implies that an individual is more likely to participate in the provision of the public goods and that this individual will contribute more. This occurs if either the cost per unit of the charitable good $q$ or the target level of consumption $c_\bar{}$ is lower, or if either the degree of altruism $\beta_i$ or the welfare increment from providing the charitable good to a beneficiary $(v_i (c) - v_i (0))$ is higher.

To prove that this equilibrium exists, simply note that the demand for private consumption $d_i$ is increasing in the cost per unit $q$ and the target level $c_\bar{}$, from which it follows that it is always possible to find a sufficiently large number of beneficiaries $K$ or a cost per beneficiary $qc_\bar{}$ such that the willingness to contribute by all donors $\sum_i \max [Y_i - d_i (V_i), 0]$ is lower than (or equal to) the cost of providing the good to all potential beneficiaries $qc_\bar{}K$. In such a situation, each contributor is doing his best given the optimal contributions of the other donors. In effect, since the level of contributions are uniquely defined by the first order conditions, the equilibrium is unique within the set of equilibria in which only a fraction of beneficiaries receive the charitable good. The following proposition summarizes this result.

**Proposition 1** If $\sum_i \max [Y_i - d_i (V_i), 0] \leq qc_\bar{}K$, there is a unique Nash equilibrium in which a subset of beneficiaries receive the charitable good and each donor contributes the amount that is individually optimal in the absence of other contributors, $\max [Y_i - d_i (V_i), 0]$.

This equilibrium has a number of additional important properties that are summarized in the following corollary.

**Corollary 1** The partial provision equilibrium has the following properties:
1. An exogenous increase in the charity’s budget (e.g. an additional contributor) increases the total level of contributions dollar-for-dollar.

2. Redistribution of income from non-contributors to contributors increases the total level of contributions dollar-for-dollar. Otherwise, redistribution is (in general) neutral.\(^{10}\)

3. A lump-sum tax on a non-contributor that is transferred to the charity (or on a contributor in excess of his equilibrium contribution) increases the total level of contributions dollar-for-dollar. Otherwise, tax changes are neutral.

In other words, altruism has no limitations in explaining the observed empirical regularities in a world in which at least some potential beneficiaries lack basic necessities. This seems to be the relevant scenario under which large charitable organizations like the American Red Cross, UNICEF, Medicins Sans Frontieres, or Feed the Children work.

### 4.2 Full provision equilibria

The partial provision equilibrium is not the only equilibrium for all parameter values. If the sum of the amounts that each donor is willing to contribute independently of the contributions of others is greater than the cost of providing the charitable good to all potential beneficiaries, the equilibrium will always involve provision to all \(K\) beneficiaries. Formally, according to Proposition 1, when \(M < K\) beneficiaries receive the charitable good, each donor \(i\) contributes \(\max [Y_i - d_i(V_i), 0]\) and the total level of contributions equals \(\sum_i \max [Y_i - d_i(V_i), 0]\). Given this level of contributions, the number of individuals that can receive the charitable good is \(M = \sum_i \max [Y_i - d_i(V_i), 0]\). However, if \(\sum_i \max [Y_i - d_i(V_i), 0] > q c K\), then \(M > K\), which is unfeasible. Moreover, it is always possible to find a set of donors that are willing to contribute exactly \(q c K\). Otherwise, a deviation from this equilibrium would imply that less than \(K\) beneficiaries receive the charitable good and that some contributors donate less than \(\max [Y_i - d_i(V_i), 0]\), contradicting Proposition 1. Of course, there are many possible configurations of contributions that equal exactly \(q c K\), so many equilibria are possible. We can then conclude,

**Proposition 2** Suppose that \(\sum_i \max [Y_i - d_i(V_i), 0] > q c K\). Then, all equilibria involve the efficient level of provision in which all \(K\) beneficiaries receive the charitable good.

\(^{10}\)A redistribution of income may not be neutral if individuals become contributors or become non-contributors after the redistribution.
Proposition 2 is substantially stronger than the predictions of other models of private provision of discrete public goods (e.g. Palfrey and Rosenthal, 1984; Bagnoli and Lipman 1989, 1992; Admati and Perry, 1991), so it is important to understand the underlying differences.

There are three closely related conditions that are necessary in order to generate the efficient outcome in which all potential beneficiaries always receive the charitable good. First, like other models of private provision of discrete public goods, discontinuities in the charitable good provided imply that additional donations may have a large impact on the level of the public good provided, making it possible to sustain high levels of contributions. Second, our assumption of a continuous measure of recipients ensures that equilibria with low levels of contributions, which are typical of other discrete public goods models, do not arise. It is simple to show, however, that if we introduce a countable set of recipients in our model (e.g. the number of disadvantaged individuals to sponsor), there may exist equilibria with levels of contributions that are below the cost of providing the minimum efficient level of the charitable good to all beneficiaries (see Section 4.3.1 below). Finally, in our model, when a donor contributes to provide the charitable good to a given recipient he does not crowd out contributions to provide the charitable good to other disadvantaged individuals. As a result, the efficient outcome with provision of a large number of "units" of charitable goods can be sustained as a Nash outcome. This is in sharp contrast to Bagnoli and Lipman’s (1989) model of multiple discrete public goods, in which the efficient outcome may not be within the set of Nash equilibria due to diminishing marginal utility over the number of units provided.

Of course, in the case of pure public commodities, such as Bagnoli and Lipman’s example of street lights, it is perfectly reasonable to presume that contributors have diminishing marginal utility over the number of units provided or, more generally, substitutability between the different public goods. However, when the public good represents the welfare of individual recipients, such assumption seems less convincing. In particular, there is no reason a-priori to believe that altruistic donors receive less satisfaction from contributing to provide a nutrition package or a vaccine to a child after another donor contributes to provide the charitable good to a different child. Our model characterizes this latter situation.
4.3 Additional considerations

4.3.1 Countable set of recipients

Let us consider the problem with a countable set of recipients. Let $\hat{K}_i$ denote the number of individuals that donor $i$ would ‘sponsor’ when acting in isolation as a single contributor and suppose that no single donor would sponsor all potential recipients. That is, $\hat{K}_i = \arg \max_{K_i \in \mathbb{N}} u_i (Y_i - qcK_i) + \beta_i [K_i v_i (c) + (K - K_i) v_i (0)]$. From Proposition 1 we know that when $\hat{K}_i \in \mathbb{R}^+$ we have $\hat{K}_i = \frac{\max [Y_i - d_i(V_i), 0]}{qc}$. Thus, given the constraint that $\hat{K}_i$ belongs to the set $\mathbb{N}$, we can conclude that $\hat{K}_i = \{\lfloor x_i \rfloor, \lceil x_i \rceil\}$, with $x_i \equiv \frac{\max [Y_i - d_i(V_i), 0]}{qc}$.

Let us assume that, in effect, $\hat{K}_i = \lfloor x_i \rfloor$ for all $i$. Moreover, let $\hat{k}_i$ denote the maximum dollar amount that donor $i$ may potentially contribute to sponsor an additional individual (when other donors contribute $qc - \hat{k}_i$) in addition to sponsoring $\hat{K}_i$ individuals -note that he would never contribute the full amount to sponsor an additional individual-. We then obtain that, 1) when $\sum \hat{K}_i > K$, the equilibrium is always efficient (i.e. all potential beneficiaries are sponsored), 2) when $\sum \hat{K}_i < K < \sum \hat{K}_i + \sum \frac{\hat{k}_i}{qc}$, the efficient equilibrium exists and the least equilibrium involves provision to $\sum \hat{K}_i$ beneficiaries, and 3) when $\sum \hat{K}_i + \sum \frac{\hat{k}_i}{qc} < K$, the least (resp. greatest) equilibrium involves provision to $\sum \hat{K}_i$ (resp. $\sum \hat{K}_i + \sum \frac{\hat{k}_i}{qc}$) beneficiaries -assuming, without loss of generality, that $\sum \frac{\hat{k}_i}{qc}$ is an integer-. Results 1) and 2) correspond to the full provision equilibrium described in Proposition 2. In contrast to Proposition 2, result 2) shows that the equilibrium may be inefficient, with only a subset of disadvantaged individuals receiving the charitable good despite the willingness to contribute by all donors being greater than the cost of sponsoring all disadvantaged individuals. Result 3) corresponds to the partial provision equilibrium described in Proposition 1. Consistent with Corollary 1, this result also implies that, starting from a partial provision equilibrium, an exogenous increase in the number of sponsored individuals will not crowd-out other contributions. In effect, an additional contribution may increase the contributions of other donors (e.g. those with $\hat{K} = 0$ but $\hat{k}_i > 0$) if, as a result, an additional beneficiary receives the charitable good.

4.3.2 Equilibria with provision above a minimum

We have so far assumed a discrete level of a charitable good that is either provided or not. For some charitable goods, a more relevant scenario is the case in which there is also a minimum efficient level of the charitable good but consumption above this level is unconstrained, so the wellbeing of the potential recipients, and by extension the altruistic
utility of donors, increases continuously above the minimum level. This alternative scenario is formally equivalent to (4) except that now potential beneficiaries consume either 0 or $c \geq \bar{c}$. The set of equilibria can then be characterized as follows. For low levels of $c$ (and of $K$ and $q$), the constraint $c \geq \bar{c}$ will be non-binding and the equilibrium will be the same as the unconstrained Nash equilibrium in the traditional analysis (i.e. when $c=0$, as in Section 3), with provision to all beneficiaries and $c = \frac{G}{qK} > \bar{c}$. For intermediate levels of $c$ ($K$ and $q$), the equilibria still involves provision to all beneficiaries and $c = \bar{c}$. In this case, contributions are higher than in the unconstrained equilibrium, but still lower than the total willingness to contribute by all donors -as in Proposition 2. Above a threshold level, say $\bar{c}^T$, the equilibrium involves provision of the minimum efficient level to only a fraction of beneficiaries and all donors contributing the amount that is individually best in the absence of other donors -as in Proposition 1.

Importantly, this alternative scenario implies that the charitable organization may play an active role in the design of the charitable goods provided in order to increase contributions. Suppose, for example, that the good under consideration is perfectly divisible, so the relevant benchmark is the unconstrained equilibrium with full provision. Then, if the charity can credibly commit to providing an amount $\hat{c}$ of the charitable good per beneficiary (or equivalently, request a minimum donation to sponsor a potential beneficiary), it can always select a level of provision that is higher than the unconstrained level of provision and lower than the threshold $\bar{c}^T$ such that, in equilibrium, all beneficiaries receive exactly $\hat{c}$.

In summary, our analytical framework based on altruistic behavior provides a solid explanation to many of the observed empirical regularities in charitable giving. In the next section, we provide a more specific application of our results.

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11 That is, there is provision to all beneficiaries as long as the inequality $\sum_i \max [Y_i - d_i (V_i), 0] > q\bar{c}K$ holds.

12 These results correspond to the case with a continuous measure of recipients. The case of a countable set of recipients can be analyzed along the lines of Section 4.3.1.

13 Importantly, in order for this commitment mechanism to be credible, the charity must be ready to leave unsponsored individuals without provision, even though this may never occur in equilibrium. Moreover, this can only occur in the context of full provision; starting from a partial provision equilibrium (i.e. when $c > \bar{c}^T$), any required contribution above $c$ would reduce total contributions (see Section 4.1).
5 An application: Disaster relief efforts

One of the most widely observed phenomena in charitable giving is that natural and man-made disasters tend to be followed by extraordinary levels of assistance among community members of the disaster-stricken areas and also from outsiders.\textsuperscript{14} In trying to explain this phenomenon, Dacy and Kunreuther (1969), the seminal work on the economics of disasters, argued that individuals may become more altruistic in the face of great adversity. De Alessi (1975) pointed out that such a shift in the utility function is not necessary to explain the increased level of assistance. Instead, he argued that, since a disaster "creates" a new group of disadvantaged individuals, altruistic individuals within and outside the disaster-stricken community may increase their contributions of time and money or they may switch their current contributions to assist the individuals that are now in most urgent need of assistance. As Hirshleifer (1983, 1987) observed, the potential difficulty for this explanation is free-riding among potential contributors.\textsuperscript{15}

Our model provides an explanation why such a concern may not be warranted. During disasters, there is usually a large number of individuals that are in immediate need of assistance, lacking some of the most basic necessities to survive. According to Proposition 2, and consistent with De Alessi’s (1975) argument, if the relevant pre-disaster benchmark is the full provision equilibrium, donations will flow into the disaster stricken area as long as the willingness to contribute by all donors remains greater than the cost of providing the essential goods and services to all potential recipients.\textsuperscript{16}

But this cannot be the end of the story. Since hundreds of thousands of individuals die every year from causes that are preventable through charitable giving, the most relevant benchmark for potentially life saving interventions of unrelated persons is the partial provision equilibrium, not the full-provision equilibrium. Thus, while any additional contribution may have a large impact in the lives of the disaster victims, the same is true for disadvantaged individuals outside the disaster-stricken area. Everything else equal,

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\textsuperscript{14}See Hirshleifer (1987) for a collection of essays on the topic.

\textsuperscript{15}As a result of this difficulty, Hirshleifer (1983, 1987) proposes an alternative explanation based on complementarities in the behavior of the community members of the disaster-stricken area (e.g. in the form of assistance to those most in need ) needed to maintain social cohesiveness. Hirshleifer (1983) argues, in particular, that in times of disaster social cohesiveness may be best described by a weakest-link aggregation rule (whereby the level of the public good provided equals the minimum of the contributions) -see also Harrison and Hirshleifer (1997)-

\textsuperscript{16}As argued in Section 3, the traditional model with multiple beneficiaries also predicts an increase in contributions when the number of potential beneficiaries increases.
an increase in the number of people dying from preventable causes (arising from a disaster) does not necessarily increase contributions when we start from a partial provision equilibrium in which there is no free riding.

According to our previous results, in order to have an increased level of assistance starting from a partial provision equilibrium, at least one of the following must be true. First, if the pre-disaster equilibrium is inefficient (see Section 4.3.1) donors may increase their contributions and sponsor additional individuals if the natural disaster generates a new equilibrium (e.g. due to news coverage of the disaster). Second, donors may switch their donations if they have a higher valuation (as determined by the cost-effectiveness measure $V_i$) for the provision of essential goods to the victims of a disaster-stricken area. There are a number of reasons to believe that this may be the case for many potential donors. Most obviously, donors that have a closer connection with the disaster's victims (e.g. due to geographic distance, nationality, ethnic ties) will likely have a greater concern over their lives (i.e. presumably, the degree of altruism towards these recipients is higher). In addition, the effectiveness of a given intervention may be perceived to be greater in some disaster-stricken areas. This may be the case, for example, if the recipients' quality-adjusted years gained for a given intervention are greater in the disaster area (e.g. higher life expectancy) or if some resources provided during disasters are not useful elsewhere (e.g. the immediate help provided by a survivor to other victims in the form of rescuing assistance or shelter). Finally, it is likely that the cost of provision per beneficiary in disaster-stricken areas is lower than the cost of provision to other areas (e.g. due to the localized nature of disasters, transportation and distribution costs may be lower).

6 Concluding remarks

We have shown that a model of charitable giving based exclusively on altruism may have more predictive power than previously understood. Given indivisibilities in the provision of charitable goods, together with a large number of potential recipients, additional donations may have a significant impact on each recipient’s welfare and, as a result, incentives to free ride may be greatly diminished. By making this fact more salient, charities that focus their fundraising campaigns on the impact of donations on each additional recipient should be more successful than charities with a broader focus (for instance, ending world hunger). While this is not unknown to fundraisers, our framework provides a simple and formal explanation why this type of information proves to be a useful marketing tool.
We emphasize that our analysis does not imply that altruism is the only reason for the observed levels of charitable giving. Without doubt, prestige and recognition by others play an important role in some philanthropic activities. The self-satisfaction received from the act of giving and or the feeling of a moral obligation to contribute to a charitable cause are also likely important for need-based charitable giving. Identifying the different motivations behind the different types of philanthropic activities would lead to a better understanding of observed behavior and improved policy-making.

Our analysis focused on contributions to a single charity and, as such, has left a number of important questions unanswered. How would our results change if potential beneficiaries have multiple complementary needs that are served by different charitable organizations? What is the optimal design of sponsorship programs, in terms of levels of provision per beneficiary, in the presence of multiple competing charities? What are the positive and normative implications of indivisibilities for the observed levels of diversification of donations among unrelated charities (e.g. sponsorship of a disadvantaged child and of a disadvantaged animal)? These questions are the subject of ongoing research.

References


\[17\] We conjecture that this possibility will strengthen our results, in the sense that strong complementarities will lead to equilibria in which some beneficiaries receive a bundle of different goods and others do not receive anything (i.e. partial provision equilibria with no free-riding). This sort of complementarities, however, are also likely to introduce additional coordination problems, with some equilibria involving very low levels of provision.


