Priority Setting in Health Care: Disentangling Risk Aversion from Inequality Aversion

*Keywords:* Risk aversion, inequality aversion, social welfare function, health uncertainty, budget allocation.
1. Introduction

How should health care resources be allocated when health outcomes are uncertain and the distribution of health outcomes differs across individuals (i.e. when there is risk and inequality)? A number of authors (e.g. Hoel, 2003; Bui et al, 2005; Courbage and Rey, 2010) have recently answered this question in the context of models where social welfare is assumed to take an expected-utility functional form. Although appealing for its connection with (individual) decision-theory, the expected utility approach to social welfare, first proposed by Harsanyi (1955), has received many criticisms. One such criticism is that it does not distinguish between attitudes towards risk and attitudes towards inequality. Consider, for instance, a version of Diamond’s (1967) famous example (see also Bleichrodt (1997) for a similar example in the context of health programs). There are two equally likely states of nature, r and s, and two individuals, A and B. The social planner is considering two alternative health policies, $P^1$ and $P^2$, which would result in the following health payoffs (net of costs):

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Under the expected utility approach the two policies would be considered equivalent since they provide the same expected utility across individuals and states of nature. Diamond argued, however, that policy $P^2$ should be socially preferable to policy $P^1$ on the grounds that the former gives group B “its fair share.” Indeed, under $P^2$ there is no inequality while under $P^1$ individual A’s expected utility is higher than that for B. However, policy $P^1$, which provides certain payoffs, is also less risky than policy $P^2$. Therefore, from a social perspective there is a trade-off between risk and inequality. The standard expected utility approach to social welfare does not capture this trade-off because it equalizes risk preferences with inequality preferences. If

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1 In the context of the evaluation of social health outcomes the utility approach was introduced by Wagstaff (1991) and further studied by Bleichrodt (1997) and Dolan (1998). These authors evaluate ex-post inequalities, that is, health differences after the states of nature are revealed. Bleichrodt et al (2008) recently considered the optimal allocation of health care resources in this setting and also in an alternative setting where equity considerations are incorporated as weights in the utility function.
Diamond is correct, however, social preferences should display a *bias* towards equality relative to risk tolerance.

The first objective of this paper is to introduce a tractable social welfare function that disentangles preferences towards risk in health outcomes from preferences towards health inequalities across individuals and is therefore able to provide a precise meaning to the mentioned trade-off. In particular, we imagine a social planner that evaluates social welfare by first calculating the certainty-equivalent health level of each individual under consideration and then aggregates outcomes (certainty-equivalents) across the population using a social welfare function that captures equity concerns. This representation allows us to establish the following intuitive fact: for a given distribution of health outcomes (either across states of nature or across the population), a policy with homogeneous but uncertain health outcomes (e.g. policy $P^2$ above) is socially preferable to a policy with heterogeneous but certain health outcomes (e.g. policy $P^1$ above) if, and only if, the degree of inequality aversion is stronger than the degree of risk aversion.

Given the proposed preference specification, the second objective of this paper is to evaluate how uncertainty affects the optimal allocation of health care resources. As Dardanoni and Wagstaff (1990) do [and more recently Nocetti and Smith (2010) and Courbage and Rey (2010)] we initially consider a linear health production function and we evaluate uncertainty over the severity of illness and over the effectiveness of treatments. For each case, we show that the separation of preferences towards risk from preferences towards inequality is critical for understanding the optimal allocation of health care resources. For example, given constant relative risk aversion, the additional amount that a social planner invests on a group whose severity of illness is uncertain increases with the degree of risk aversion but *decreases* with the degree of inequality aversion. Furthermore, whether uncertainty over the effectiveness of treatments increases or decreases the optimal investment depends exclusively on the degree of inequality aversion; the degree of risk aversion may decrease the optimal investment. This is in sharp contrast with the standard expected utility approach [e.g. Courbage and Rey (2010)], which predicts that under both types of risks the social planner would invest more given a higher degree of risk aversion or, equivalently, a higher degree of inequality aversion. We also show that in a society that is highly averse to health inequalities, which we call a “Rawlsian” society, risk
aversion [instead of prudence as in Hoel (2003), Bui et al (2005), and Courbage and Rey (2010)] is necessary and sufficient for a higher investment when there is uncertainty of either sort.

We then extend our analysis to evaluate a more general production of health in the spirit of Hoel (2003) and Bui et al (2005). Not surprisingly, the conditions for a larger investment in the group with an uncertain health status in this more general case are more complex than in the linear case. In particular, the conditions depend not only on social preferences towards risk and towards inequality but also on the curvature of the health production function and on the interaction between the marginal productivity of public investments and the stochastic component of health.

The paper proceeds as follows. The next section introduces the preference structure that will be used throughout the paper, provides some justifications for the representation proposed, and reconsiders Diamond’s (1967) example within this framework. In Section 3 we analyze the optimal allocation of resources in the context of a linear production function of health and risks surrounding the severity of sickness and the efficiency of treatments. In Section 4 we consider the case of a more general production function. Our concluding remarks are presented in section 5, while all proofs and derivations are in the Appendix.

2. Social Preferences

Consider a population of $Z$ individuals and let us assume that health can be quantified, for example, by quality adjusted life years. Health for an individual $i$ ($i = 1, 2, \ldots, Z$) is denoted $H_i$, which may be stochastic. We further assume that the decision-maker evaluates social welfare by proceeding in two stages:

First, given a von-Neumann-Morgenstern expected utility function defined over the random health of individual $i$, $E\nu(H_i)$, the decision-maker calculates the certainty-equivalent level of health for this individual; that is, the certain amount of health, say $H_i^{CE}$, that provides the individual under consideration the same utility as the expected utility of his or her random health,

$$H_i^{CE} = \nu^{-1}[E\nu(H_i)].$$
As usual, the curvature of $v(.)$ measures the attitude towards risk faced by this individual. The Arrow-Pratt index of absolute risk aversion is $R^a = \frac{v''(x)}{v'(x)}$ and the index of relative risk aversion is $R^r = -\frac{v''(x)}{v'(x)} x$.

Second, the decision-maker evaluates social outcomes by using the following social welfare function (SWF)

$$W(H_1^{CE}, H_2^{CE}, \ldots, H_Z^{CE}) = \sum_{i=1}^{Z} U(H_i^{CE}).$$

That is, the SWF is defined as the average utility (across the population) of the certainty-equivalent health levels. The curvature of $U(x)$ captures social attitudes towards unequal distributions of risky health prospects across the population. In particular, we will say that social preferences display inequality aversion if a homogeneous population with a certainty-equivalent health level $\sum_{i=1}^{Z} H_i^{CE}$ is preferred to a heterogeneous population in which individual $i$ has a certainty-equivalent health level $H_i^{CE}$, i.e. if $U\left(\sum_{i=1}^{Z} H_i^{CE}\right) \geq \sum_{i=1}^{Z} U(H_i^{CE})$. Clearly, this is true if $U''(x) < 0$. We will therefore refer to $I^a = -\frac{U''(x)}{U'(x)}$ as the index of absolute inequality aversion and to $I^r = -\frac{U''(x)}{U'(x)} x$ as the index of relative inequality aversion.

**Remark.** When only ex-post outcomes are analyzed (i.e. health levels after the state of nature is revealed), it is natural to define inequality aversion with respect to differences in individual health levels (e.g. Wagstaff, 1991; Dolan, 1998; Bleichrodt et al 2008). So, for example, concavity of the utility function implies that it is preferable to increase the health level of an individual with poor health rather than the health level of an individual in good health.\(^2\) Our definition is rather an ex-ante notion of inequality, in the sense that it refers to health levels before the state of nature is revealed and, as such, it is related to differences in the distributions of risky health outcomes faced by individuals. In particular, in our setting concavity of $U$ implies that it is preferable to increase the “units” of certainty-equivalent health levels of individuals

\(^2\) This is precisely what our representation implies in the absence of risk.
with low (rather than high) certainty-equivalent health.\footnote{The expected utility model does not imply indifference towards ex-ante inequalities, but, as it is clear from Diamond’s (1967) example, it does not differentiate inequalities across individuals from inequalities across states of nature.} Our interpretations below provide further justification for our definitions. ■

Eq. (2) includes the following representations as special cases:

- The standard expected utility SWF (Harsanyi, 1955), which arises when \( v = U \equiv u \), so
  \[
  W(H_1^{CE}, H_2^{CE}, \ldots, H_Z^{CE}) = \sum_{z}^{1} u(H_i),
  \]  

- The case of inequality neutrality, which arises if \( U \) is linear, so the SWF is represented as the average of certainty equivalents,
  \[
  W(H_1^{CE}, H_2^{CE}, \ldots, H_Z^{CE}) = \sum_{z}^{1} \frac{1}{Z} H_i^{CE}.
  \]  

- The case of extreme inequality aversion, which we will denote Rawlsian preferences [Rawls (1971, 1974)], where social preferences only depend on the least well-off individual; that is, the individual with the lowest certainty-equivalent health level,\footnote{Eq. (5) is ordinal equivalent to a CES function. It is obtained in the limit as the (constant) degree of relative inequality aversion (i.e. the reciprocal of the elasticity of substitution) goes to infinity. The more general CES specification has been used previously in the literature in non-stochastic settings [e.g. Johannesson and Gerdtham (1996), Dolan (1998)].}
  \[
  W(H_1^{CE}, H_2^{CE}, \ldots, H_Z^{CE}) = \text{Min}(H_1^{CE}, H_2^{CE}, \ldots, H_Z^{CE})
  \]  

At least two possible interpretations can be given to our representation of social preferences. First, we can think of \( v(H) \) as the utility function over risky health prospects of the individuals under consideration. In order to evaluate social welfare the decision-maker first needs an interpersonally comparable notion of utility. As argued by Chambers (2010), a natural benchmark for comparability in the context of risky prospects is the certainty-equivalent. According to (3), the decision maker indeed uses this benchmark and then aggregates social outcomes using a standard social welfare function which captures his attitudes towards inequality.
It has been argued, however, that social choices should be evaluated by considering only the preferences of the individuals affected by those choices. Our second interpretation satisfies this requirement and relies on Harsanyi’s (1953, 1955) idea of a decision-maker acting behind the veil of ignorance (see also Rawls (1971, 1974)). Harsanyi (1953, 1955) starts by assuming that individuals evaluate risky prospects according to the axioms of expected utility. Then, to evaluate social choices, Harsanyi imagines a decision-maker acting behind the veil of ignorance, in the sense that the decision-maker eventually acquires the identity of one of the individuals under consideration, but at the point of making the choice he does not know which identity he will take. That is, the decision maker faces two types of risky prospects, one about his identity and one about the risky prospects faced by each of the individuals under consideration. The punch line of Harsanyi’s story is that if individuals act according to the axioms of expected utility, risky social prospects should also be evaluated with an expected utility functional (i.e. Eq. (3)).

To justify Eq. (2), let us maintain the story of the decision-maker acting behind the veil of ignorance, but suppose that the individuals under consideration have second-order expected utility preferences à la Klibanoff et al. (2005). In particular, notice that we can think of a decision-maker acting behind the veil of ignorance as facing ambiguity over his health prospects, in the sense that he evaluates multiple distributions without knowing which one is the “true” distribution (i.e. which one is his true identity). The expected utility model treats such uncertainty over distributions no different from uncertainty over outcomes. Instead, it has been shown that decision-makers frequently display a stronger distaste over ambiguous prospects. In response to this empirical regularity, Klibanoff et al. (2005) proposed an alternative to the expected utility approach. In their model a decision-maker facing multiple probability distributions evaluates welfare by proceeding in two stages. First, the decision-maker uses a von-Neumann-Morgenstern utility function \( v \) and “first order” probabilities \( p_{ij} \) that an outcome \( j \) will occur under distribution \( i \) to calculate the conditional certainty-equivalent for each distribution, 

\[
CE_i = v^{-1}\left[\sum_j p_{ij}v(x_{ij})\right].
\]

Then, the decision-maker uses a von-Neumann-Morgenstern utility function \( U \) and “second order” probabilities \( q_i \) (i.e. the probability that \( i \) is the true probability distribution) to calculate the expected utility of the certainty-equivalents established in the first

\[5\] One of the best examples of this regularity is Ellsberg’s paradox (Ellsberg, 1961)
stage, $\sum_i q_i U\{CE_i\}$. This representation has exactly the same form as Eq. (2). In other words, according to this interpretation, our decision-maker assigns a probability $q_i = \frac{1}{Z}$ that distribution $i$ is the true distribution (i.e. of acquiring the identity of individual $i$) and, in the face of such ambiguity, he evaluates social prospects according to the axioms of second-order expected utility.

To complete our story it is important to link our interpretation of the curvature of the functions $\nu$ and $U$ with the interpretation provided by Klibanoff et al. They suggest, as we do, that the curvature of the function $\nu$ has the standard interpretation of capturing attitudes towards risk. Importantly, however, aversion to inequality is not equivalent to aversion to ambiguity. Inequality aversion ($U$ concave) implies that the decision-maker acting behind the veil of ignorance dislikes fair lotteries over unequal identities. Instead, ambiguity aversion reflects a decision-maker’s attitude towards ambiguous prospects relative to the attitude towards the same prospects of an expected utility maximizer. In particular, as proposed by Klibanoff et al, it is natural to call an individual ambiguity averse if the function $U$ is more concave than the function $\nu$, $-\frac{\nu''}{\nu'} > -\frac{U''}{U'}$. That is, our decision-maker is ambiguity averse if his degree of inequality aversion is stronger than his degree of risk aversion, $I^a > R^a$.7

We are not dogmatic about any one of these two interpretations. Instead, we view Eq. (2) as an intuitive and well-grounded representation of social preferences that is flexible enough to disentangle two key aspects of preferences that, we believe, need not be equalized.

To begin understanding the importance of the distinction between preferences towards risk and preferences towards inequality, consider again Diamond’s (1967) example presented in the introduction. As mentioned there, under the standard expected utility SWF (Eq. 3) the two

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6 See also Nau (2006, Section 5) and Neilson (2010) for equivalent representations of second-order expected utility. As argued by Klibanoff et al (2005), the well-known maxmin expected utility model of Gilboa and Schmeidler (1989) arises as a special case of their representation. See also Gollier (2008, 2009) for two recent applications of second-order expected utility.

7 Intuitively, under the standard expected utility approach, in which $U = \nu$, identity lotteries are disliked as much as lotteries over outcomes. Instead, ambiguity aversion implies that the former are perceived as more harmful. An analogy may help to fix ideas. Kreps and Porteus (1978) proposed a similar representation of preferences in the context dynamic stochastic models. In their model, concavity of the function $\nu$ captures risk aversion, concavity of the function $U$ captures resistance to intertemporal substitution, while the condition $I^a > R^a$ implies a preference for early resolution of uncertainty.
policies would be considered equivalent. Instead, under Rawlsian preferences (Eq. 5) policy $P^2$ is socially preferable while under inequality neutrality (Eq. 4) policy $P^1$ is socially preferable. The SWF given in Eq. (2) provides a more general view of this trade-off: Policy $P^2$ is socially preferable to policy $P^1$ if, and only if, the degree of inequality aversion is stronger than the degree of risk aversion, $I^a > R^a$ (i.e. under ambiguity aversion given our previous interpretation). More generally, we can prove the following result (See Appendix).

**Proposition 1.** Suppose that social welfare is evaluated as in Eq. (2) and consider two policies, $P^x$ and $P^y$, and a random variable $\tilde{H}$. Suppose that under policy $P^x$ there is no uncertainty about health outcomes and $\tilde{H}$ represents the distribution of health across the population. Under policy $P^y$ there is no inequality and the health for each individual is characterized by the random variable $\tilde{H}$. Then, the latter is preferred to the former if and only if the degree of absolute inequality aversion is greater than the degree of risk aversion, i.e. $P^y \succ P^x$ if, and only if, $I^a \geq R^a$.

This Proposition states precisely the intuitive condition under which social preferences will display a bias towards equality of health outcomes across the population relative to tolerance of risks faced by individuals. As we shall demonstrate, the condition $l^a > R^a$ also plays an important role in the optimal allocation of health care resources.

3. **Optimal Allocations**

Consider a social planner trying to allocate a fixed budget of size $W$ to improving the health of $Z$ individuals. We will assume, without much loss of generality, that the population is composed by two groups, A and B, with shares $s_A$ and $s_B$, respectively. Let $H_i(c_i)$ be the, possibly stochastic, level of health for an individual in group $i$ ($i=A,B$) when $c_i$ is spent on a per capita basis. In this section we will start with a simple case and assume, as Dardanoni and Wagstaff (1990) do, that the health production function is linear

\[
H_i(c_i) = \mu_i c_i + \chi_i,
\]

Our results would not change if we assume that the health production function takes the form $H_i(c_i) = \mu_i f(c_i) + \chi_i$ and investments in health have decreasing returns (i.e. $f'' < 0$). The case of a more general production function is important and we consider it in section 4.
where \( \chi_i \) represents the basic level of health (alternatively, the severity of sickness) while \( \mu_i \) determines the efficiency of medical treatments. The social planner’s problem is

\[
\max_{c_A, c_B} \sum s_i U\left( H_i^{CE}(c_i) \right) \quad \text{s.t.} \sum s_i c_i = w (= W / Z),
\]

(7)

where, to recall, \( H_i^{CE}(c_i) = \nu^{-1}[Ev(H_i(c_i))] = \nu^{-1}[Ev(\mu_i c_i + \chi_i)] \).

The optimality condition is

\[
U\left( H_A^{CE}(c_A^*) \right) \frac{\partial H_A^{CE}(c_A^*)}{\partial c_A} = U\left( H_B^{CE}(c_B^*) \right) \frac{\partial H_B^{CE}(c_B^*)}{\partial c_B}
\]

(8)

where \( \frac{\partial H_i^{CE}(c_i^*)}{\partial c_i} = \frac{Ev(\mu_i c_i + \chi_i) \mu_i}{\nu(\nu(c_i^*))} \).

We will assume that the second order condition is satisfied. In the following sections we evaluate how optimal allocations change in the presence of 1) uncertainty surrounding the severity of illness (which we denote health risks) and 2) uncertainty surrounding the efficiency of medical treatments (which we denote medical care risks).

3.1. Health risks

Suppose that the two groups have the same efficiency of treatments \( \mu \) but the severity of the disease for group A is uncertain,

\[
\chi_A = \bar{\chi}_A + \epsilon,
\]

(9)

where \( \epsilon \) is a mean-zero random variable with variance \( \sigma_\epsilon^2 \). Let \( c_A^* \) and \( c_A^{cert} \) denote the optimal investments in group A with and without the risk, respectively. The addition of the risk \( \epsilon \) increases the optimal investment in group A if the marginal benefit in the presence of the risk is larger than in the absence of it, which implies the following inequality.
Borrowing terminology from Kimball (1990) and Kimball and Weil (2009), we will therefore define the “equivalent health-precautionary-premium” (precautionary premium for short) \( \psi \) as follows

\[
U\left( H_A^{CE}(c_A^*) \right) \frac{\partial H_A^{CE}(c_A^*)}{\partial c_A} = U'\left( \mu c_A^* + \overline{\mu} \right) \mu. \tag{10}
\]

The precautionary premium \( \psi \) is the certain decrease in the level of health that has the same effect on the optimal investment as the addition of the risk \( \varepsilon \). That is, in both cases, with the addition of the risk \( \varepsilon \) and with the certain change in health \(-\psi\), the optimal investment changes from \( c_A^{cert} \) to \( c_A^* \) (Kimball, 1990). Clearly, investment is higher in the presence of the risk if the premium is positive, \( \psi > 0 \). If, in addition, group B has a basic level of health equal to \( \overline{\lambda}_A \), we can also conclude that \( c_A^* > c_B^* \) if, and only if, \( \psi > 0 \).

Consider first the case in which the risk \( \varepsilon \) is small. A local approximation of \( \psi \) is given by (See Appendix)

\[
\psi \approx R^a (1 + \omega / I''') \frac{\sigma^2}{2}, \tag{12}
\]

where \( \omega = -\frac{R^a}{R^a} \) is the elasticity of risk tolerance. Clearly, \( \psi > 0 \) if and only if \( 1 + \omega / I''' > 0 \).

This has a number of important implications, which also hold for large risks (See Appendix):

**Proposition 2.** Suppose that the two groups have the same efficiency of treatments but the severity of the disease for group A is uncertain. Then,

1. A sufficient condition for the precautionary premium as defined in Eq. 11 to be positive is 
   a) \( I^a > R^a \) and b) \( v''' > 0 \). Either a) or b) is necessary. Therefore, \( v''' > 0 \) is necessary and sufficient if \( I^a = R^a \) and \( I^a > R^a \) is necessary and sufficient if \( v''' = 0 \).

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9 In particular, the optimality condition under certainty is \( U'(\mu c_A^{cert} + \overline{\lambda}_A) - U'(\mu c_B(c_A^{cert}) + \overline{\lambda}_B) = 0 \). Concavity of \( U \) implies that \( c_A^* \geq c_A^{cert} \iff U'(\mu c_A^* + \overline{\lambda}_A) - U'(\mu c_B(c_A^*) + \overline{\lambda}_B) \leq 0 \). Since the first order condition under uncertainty implies \( U'(H_A^{CE}(c_A^*)) \frac{\partial H_A^{CE}(c_A^*)}{\partial c_A} - U'(\mu c_B(c_A^*) + \overline{\lambda}_B) = 0 \) and the second order condition is assumed to hold, we have that \( c_A^* \geq c_A^{cert} \) is equivalent to condition (11).
2. Irrespective of the level of $I^a$, another sufficient condition for the premium to be positive is decreasing absolute risk aversion, $\omega > 0$, which is also necessary if the social planner is inequality neutral ($I^a \rightarrow 0$).

3. With Rawlsian preferences ($I^a \rightarrow \infty$) risk aversion is necessary and sufficient for a positive precautionary premium.

4. Given decreasing absolute risk aversion, the precautionary premium decreases with the degree of inequality aversion.

Proposition 2 generalizes the results of Hoel (2003) and Bui et al. (2005) who showed that, under the expected utility approach, the condition $v''' > 0$ (prudence) is necessary and sufficient for a larger investment in the group that has a riskier health profile. Under our more general preferences $v''' > 0$ is still sufficient if the degree of inequality aversion is stronger than the degree of risk aversion. As we showed above, this would be the case given Diamond’s suggestion that policy $P^2$ should be socially preferable to policy $P^1$ (equivalently, if a decision-maker acting behind the veil of ignorance is ambiguity averse). The Proposition further shows that in the case of Rawlsian preferences only risk aversion is necessary and sufficient for a larger investment and that, without restricting the degree of inequality aversion, the more stringent but reasonable condition that risk aversion is decreasing is sufficient.

As stated in the fourth part of the Proposition, the fact that the precautionary premium is positive under weaker conditions when the degree of inequality aversion is high should not be confused with the precautionary premium increasing with the degree of inequality aversion. In fact, if absolute risk aversion is decreasing the opposite is true: the more inequality averse the social planner is, the lower the additional investment in the group with a risky health profile will be. This is in sharp contrast with the expected utility approach which predicts that, for a given level of $\omega$ (e.g. $\omega = 1$ under constant relative risk aversion), the social planner would invest more given a higher degree of inequality aversion.

The intuition for this result is that, given decreasing absolute risk aversion, the social planner will allocate more resources to the group with a risky health profile than the amount of resources required to compensate for a decrease in utility (i.e. the precautionary premium is larger than the risk premium). However, this additional compensation increases the degree of
inequality between the two groups. The more inequality averse the social planner is, the lower the additional compensation. Therefore, for example, under extreme inequality aversion (or when absolute risk aversion is constant) there is no extra compensation and the precautionary premium equals the risk premium.

**Remark.** These results are closely related to those of Kimball and Weil (2009). In a two period model of saving, and using a preference specification that separates risk aversion from preferences towards intertemporal substitution [Selden (1978), Kreps and Porteus (1978)] they showed that, given decreasing absolute risk aversion, 1) the precautionary saving premium is positive and 2) the precautionary saving premium decreases with the resistance to intertemporal substitution. See also Nocetti and Smith (2010, 2011) ■

### 3.2. Medical care risk

Consider now the case in which the health production functions are \( H_i = \mu_i c_i \ i = A, B, \) and there is uncertainty surrounding the efficiency of treatments for group A, so \( \mu_A = \mu \hat{\epsilon} , \) where \( \hat{\epsilon} \) is a mean-1 random variable.\(^{10}\) In a similar fashion to the case of health risk, we define the proportional equivalent health-precautionary-premium \( \hat{\psi} \) as follows

\[
U' \left( H_A^CE(c_A^*) \right) \frac{\partial H_A^CE(c_A^*)}{\partial c_A} = U' \left( \mu c_A^* \left( 1 - \hat{\psi} \right) \right) \mu
\]

The precautionary premium \( \hat{\psi} \) is the certain proportional reduction of health (e.g. the certain reduction in medical care efficiency) that has the same effect on the optimal investment as the introduction of the proportional risk \( \hat{\epsilon} \). If \( \hat{\psi} > 0 \) then, by concavity of the utility function, the risk increases the optimal investment in group A relative to the case of no risk. If, in addition, group B’s efficiency of treatments equals the expected efficiency of group A, we can also conclude that \( c_A^* > c_B^* \) if, and only if, \( \hat{\psi} > 0 \).

\(^{10}\) Notice that we assumed that the basic level of health \( \chi_i \) is zero. Considering the case of a positive basic level of health would not alter our results significantly, but it would require introducing new notation in terms of partial measures of inequality aversion and of risk aversion (e.g. \( R^p = -\frac{\nu''(x+y)}{\nu'(x+y)} x \) (on partial measures of risk aversion see e.g. Chiu et al. 2010).
To provide intuition for our more general results, let us first derive a local approximation of \( \hat{\psi} \). We have (See Appendix)

\[
\hat{\psi} \approx R^r \left(1 + \frac{(\omega - 2)}{I^r} \right)^{\frac{\sigma_x^2}{2}}
\]

Eq. 14 implies that the local precautionary premium is positive if, and only if, we have \( I^r + \omega > 2 \). As before, both attitudes towards risk and attitudes towards inequality play a crucial role to determine the effect of risk surrounding the effectiveness of treatments on the allocation of public health investments. The following Proposition provides a closer look at the properties of \( \hat{\psi} \) for both small and large risks.

**Proposition 3.** When there is uncertainty in the efficiency of treatments for group A,

1. A sufficient condition for the precautionary premium \( \hat{\psi} \) as defined in Eq. 14 to be positive is a) \( I^a > R^a \) and b) \( -\frac{\varphi(r(x))}{\psi(r(x))} \times > 2 \). Either a) or b) is necessary.

2. Another sufficient condition for the premium to be positive is decreasing relative risk aversion, which implies \( \omega > 1 \), and \( I^r \geq 1 \). Given constant relative risk aversion, \( I^r > 1 \) is necessary and sufficient for a positive premium.

3. With Rawlsian preferences \( (I^a \rightarrow \infty) \) risk aversion is necessary and sufficient for a positive premium.

Part 1 of the Proposition generalizes the result of Courbage and Rey (2010), who showed that under the expected utility approach an increase in risk in the efficiency of treatments increases the optimal investment if, and only if, relative prudence is larger than two. Under our more general preferences this condition is still sufficient if the degree of inequality aversion is stronger than the degree of risk aversion. But this condition is no longer necessary. In fact, under Rawlsian preferences only risk aversion is necessary (and sufficient) for uncertainty to increase the optimal investment.

Possibly the more interesting result is the fact that under constant relative risk aversion only the degree of aversion to inequality matters to determine whether uncertainty surrounding the efficiency of treatments increases or decreases the optimal investment. The degree of risk aversion only affects the magnitude of the premium, decreasing the optimal investment if
inequality aversion is sufficiently low. This is in sharp contrast with the expected utility approach, which predicts that an increase in risk aversion should increase the precautionary premium. In fact, if \( I^r = 1 \) (log inequality preferences), and whatever the degree of risk aversion, the optimal investment does not change at all in the presence of uncertainty surrounding the efficiency of treatments.

4. A More General Production Function

Hoel (2003) and Bui et al (2005) consider a more general model for the production of health. Let us then follow their example and assume that health status for group \( i \) depends on two variables, health investments \( c_i \) and a random variable \( \theta_i: H(c_i, \theta_i) \). We assume that \( H_c > 0 \), \( H_{cc} \leq 0 \), and \( H_\theta > 0 \). Eq. (7) in the text is clearly a special case of this formulation if we interpret \( \theta \) as capturing either the severity of disease or the efficiency of treatment.

As we did before, we want to compare the optimal level of investment in a given group, say group A, in the presence of a mean-zero risk \( \varepsilon \) added to \( \theta_A \) relative to the case of no risk. We point out that this is different from the approach of Hoel (2003) and Bui et al (2005), who consider as a benchmark the optimal investment when the health status of the group equals \( EH(c_A, \theta_A + \varepsilon) \) instead of our benchmark with no risk (see remark below). The same arguments used above imply that investment will be higher in the risky scenario if the marginal utility of investment is greater in this case:

\[
U'(v^{-1}[Ev(H(c_A^*, \theta_A + \varepsilon))]) \frac{Ev'(H(c_A^*, \theta_A + \varepsilon)H_c(c_A^*, \theta_A + \varepsilon))}{v'(v^{-1}[Ev(H(c_A^*, \theta_A + \varepsilon))])} \geq U'(H(c_A^*, \theta_A))H_c(c_A^*, \theta_A)
\]

(15)

This condition is satisfied if (see Appendix):

- \( I^a \geq R^a \) and
- \( v'(H)H_c \) is convex in \( \theta \), a condition which can be written as

\[
v'''H_cH_\theta^2 + 2v''H_\theta H_{c\theta} + v'''H_cH_{\theta\theta} + v'H_{c\theta\theta} > 0
\]

(16)
Part 1 of Proposition 2 arises as a special case in which \( H_{c\theta} = H_{\theta\theta} = H_{c\theta\theta} = 0 \), while part 1 of Proposition 3 arises as a special case in which \( H_\theta = H_{c\theta} = 1 \) and \( H_{\theta\theta} = H_{c\theta\theta} = 0 \).

Clearly, the curvature of the health production function and the dependence of the marginal productivity of investments on the random variable make the analysis more complex than before. For example, an interesting scenario not analyzed before is the case in which the severity of the disease changes the marginal productivity of health investments. It is reasonable to presume that the marginal productivity of health investments decreases at higher levels of basic health, so \( H_{c\theta} < 0 \) (i.e. investments and basic health are substitutes) and that preferences display risk aversion and prudence. Then, if health status is concave in the basic level of health, \( H_{\theta\theta} < 0 \), and if the marginal productivity of investments is convex in the basic level of health, \( H_{c\theta\theta} > 0 \) (i.e. the presence of the risk decreases the expected level of health and increases the marginal product of public investments), then, condition 16 will hold.\(^{11}\) If, in addition, the degree of inequality aversion is at least as high as the degree of risk aversion, \( I^a \geq R^a \), uncertainty in the basic level of health will increase the optimal investment.\(^{12}\)

**Remark.** Hoel (2003) and Bui et al (2005) compare the level of investment in a given group in the presence of a mean zero risk \( \varepsilon \) relative to the investment in this group when their health status equals \( EH(c, \theta + \varepsilon) \). We believe that our approach is neither superior nor inferior to theirs; instead, the two cases provide different benchmarks. They show, in particular, that risk aversion and prudence, together with \( H_{c\theta} < 0 \), imply a higher investment in the group with risky health status (as we argued above, \( H_{c\theta} < 0 \) is reasonable if we think of \( \theta \) as the severity of illness). It is simple to show that in our framework, but with this different benchmark, these conditions are still sufficient for a higher investment if inequality aversion is stronger than risk aversion as suggested by Diamond’s (1967) example.\(\blacksquare\)

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\(^{11}\) For example, if \( H(c, \theta) = \theta + \mu(c + b\theta)^\alpha \), with \( 0 < \alpha \leq 1 \), then condition (16) will be satisfied. The first term, \( \theta \), can be interpreted as the basic level of health while the second term, \( \mu(c + b\theta)^\alpha \), can be interpreted as the production of additional health. Eq. 6 arises as a special case in which \( b = 0 \) and \( \alpha = 1 \).

\(^{12}\) Consistent with Proposition 1, it is also possible to show that investment will be higher in the presence of the risk if preferences display extreme inequality aversion and the risk decreases expected utility (e.g. \( \nu^\gamma \leq 0 \) and \( H_{\theta\theta} < 0 \)) or if a properly defined risk premium is positive and decreasing in the level of investment.
5. Conclusion

In this paper we presented a social welfare function that disentangles preferences towards risk across health outcomes from preferences towards inequalities across individuals in the population. We showed that such distinction is critical for evaluating the desirability of different health policies and, in particular, for establishing the optimal allocation of health care resources. We believe that the applicability of the proposed specification is much broader and it lends itself to any problem that involves the evaluation of social welfare in the presence of both risk and inequality.

While we believe that our representation of social preferences is appealing, it is not the only possible approach to separate risk and inequality attitudes. For example, in the context of the evaluation of ex-post health outcomes, Bleichrodt et al. (2004) propose the rank-dependent QALY model as an alternative to the utility approach of Wagstaff (1991) and Dolan (1998). Under the rank-dependent QALY model, equity considerations are incorporated through a weighting function of health outcomes, with inequality aversion reflected by higher weights given to those individuals in relatively poor health. Bleichrodt et al (2008) study the optimal allocation of health care resources under the two different approaches and conclude that each approach has relative strengths and weaknesses over the other. It would be interesting to introduce ex-ante evaluations and explicitly model risk attitudes in the rank-dependent QALY model (e.g. through a rank-dependent weighting function of certainty-equivalent health levels) and to compare the implications for priority setting in health care of such an approach with those in this paper.

Another interesting extension of our work would be to consider a preference structure that further disentangles preferences towards ex-ante inequality from preferences towards ex-post inequality. That is, while in our model risk and inequality are distinguished, our proposed preference specification does not take into account ex-post inequality. We conjecture that, in the spirit of Ben Porath et al (1997), a social welfare function that is a linear combination of our specification and the certainty equivalent of the equally-distributed ex-post health levels will

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13 More precisely, in the rank-dependent QALY model, Bleichrodt et al. (2004) give equity weights to expected health levels (expected QALYs), so it can potentially incorporate ex-ante evaluations, but it presumes risk neutrality with respect to QALYs.
achieve such objective. The optimal allocation of health care resources in such a framework is a further interesting line of inquiry for future research.

6. Appendix

6.1. Proof of Proposition 1

Under policy $P^x$ social welfare is given by $EU(\bar{H})$ while under policy $P^y$ social welfare is given by $U(v^{-1}[Ev(\bar{H})])$. Therefore, $P^y \succeq P^x$ if, and only if, $U(v^{-1}[Ev(\bar{H})]) \geq EU(\bar{H})$, or equivalently, $v^{-1}[Ev(\bar{H})] \geq U^{-1}\left(EU(\bar{H})\right)$. But this is equivalent to evaluating whether the certainty equivalent under utility function $v$ is larger or smaller than the certainty (equally distributed) equivalent under $U$. It is well known that an increase in risk aversion decreases the certainty equivalent (e.g. Gollier 2001 pg 20). Therefore $v^{-1}[Ev(\bar{H})] \geq U^{-1}\left(EU(\bar{H})\right)$ holds if, and only if, $v$ is less concave than $U$, i.e. if $I^a \geq R^a$.

6.2. Small risk approximation of the precautionary premium under health risk

Recall that the precautionary premium under health risk is defined by the equation

$$U\left(H^CE(\bar{c}_A)\right) \frac{\partial H^CE(c_A^*)}{\partial c_A} = U\left(\mu c_A^* + \bar{\lambda}_A - \psi\right) \mu, \quad \text{(A.1)}$$

with $H^CE(c_A^*) = v^{-1}\left(Ev(\mu c_A^* + \bar{\lambda}_A + \epsilon)\right)$. Let $h^* \equiv \mu c_A^* + \bar{\lambda}_A$. A second order approximation of $Ev(\mu c_A^* + \bar{\lambda}_A + \epsilon)$ around $h^*$ yields

$$Ev(\mu c_A^* + \bar{\lambda}_A + \epsilon) = v(h^*) + v''(h^*) \frac{\sigma_e^2}{2} + o(\sigma_e^2).$$

where $o(\sigma_e^2)$ collects terms that go to zero faster than $\sigma_e^2$. Therefore,

$$H^CE(c_A^*) = v^{-1}\left(v(h^*) + v''(h^*) \frac{\sigma_e^2}{2} + o(\sigma_e^2)\right).$$
Therefore, as shown by Pratt (1964), for small risk $\sigma^2_\varepsilon$ the certainty-equivalent can be approximated by

$$H^*_A(c^*_A) = v^{-1}(v(h^*)) + v^{-1}(v(h^*))v''(h^*) \frac{\sigma^2_\varepsilon}{2} = h^* + \frac{v''(h^*)\sigma^2_\varepsilon}{v(h^*)} = h^* - R^a(h^*) \frac{\sigma^2_\varepsilon}{2} + o(\sigma^2_\varepsilon)$$

\[(A.2)\]

Differentiating $A.2$ with respect to $c_A$ we obtain

$$\frac{\partial H^*_A}{\partial c_A} = \mu \left(1 - R^a(h^*) \frac{\sigma^2_\varepsilon}{2}\right) + o(\sigma^2_\varepsilon)$$

\[(A.3)\]

Also using $A.2$, a first order approximation of $U\left(H^*_A(c^*_A)\right)$ around $h^*$ yields

$$U\left(H^*_A(c^*_A)\right) \approx U(h^*) - U''(h^*) R^a(h^*) \frac{\sigma^2_\varepsilon}{2} + o(\sigma^2_\varepsilon)$$

\[(A.4)\]

Using $A.3$ and $A.4$ we can write the left hand side of $A.1$ as

$$\left[U'(h^*) - U''(h^*) R^a(h^*) \frac{\sigma^2_\varepsilon}{2} + o(\sigma^2_\varepsilon)\right] \left[\mu \left(1 - R^a(h^*) \frac{\sigma^2_\varepsilon}{2}\right) + o(\sigma^2_\varepsilon)\right]$$

\[(A.5)\]

Finally, a first order approximation of the right hand side of $(A.1)$ around $\psi = 0$ yields

$$U'(h^* - \psi) \mu \approx (U'(h^*) - U''(h^*)\psi) \mu$$

\[(A.6)\]

Setting $A.5$ equal to $A.6$, as required by $A.1$, we obtain

$$\left[U'(h^*) - U''(h^*) R^a(h^*) \frac{\sigma^2_\varepsilon}{2} + o(\sigma^2_\varepsilon)\right] \left[\mu \left(1 - R^a(h^*) \frac{\sigma^2_\varepsilon}{2}\right) + o(\sigma^2_\varepsilon)\right] = (U'(h^*) - U''(h^*)\psi) \mu$$

\[(A.7)\]

Solving for the precautionary premium yields\(^\dagger\)

$$\psi = \left(\frac{U'}{U''} R^a + R^a\right) \frac{\sigma^2_\varepsilon}{2} + o(\sigma^2_\varepsilon),$$

\[(A.8)\]

\(^\dagger\) Notice that the terms including $\sigma^4_\varepsilon$ are part of $o(\sigma^2_\varepsilon)$. 

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Finally, given the definitions of relative inequality aversion, $I^R = -\frac{\nu'(x)}{\nu(x)} x$, and the elasticity of risk tolerance $\omega = -\frac{R^a(x)}{R^a(x)} x$ the expression for the risk premium can be rewritten as Eq. 12 in the text.

### 6.3. Proof of Proposition 2

1. To prove the first part of Proposition 2 we begin by re-writing social welfare as follows

$$\sum s_i U \left( H_i^{CE}(c_i) \right) = \sum s_i g \left( Ev(h_i^* + \epsilon) \right)$$

(A.9)

where $g(x) \equiv U \left( v^{-1}(x) \right)$. Eq. 12 can then be written equivalently as

$$g' \left( Ev(h^* + \epsilon) \right) Ev'(h^* + \epsilon) = U'(h^* - \psi)$$

(A.10)

By Jensen’s inequality, $v''' > 0 \iff Ev'(h^* + \epsilon) > v'(h^*)$. Furthermore, risk aversion implies that $Ev(h^* + \epsilon) < v(h^*)$. If $g$ is concave we must then have $g' \left( Ev(h^* + \epsilon) \right) > g' \left( v(h^*) \right)$. Therefore, $v''' > 0$ and concavity of the function $g$ implies

$$g' \left( Ev(h^* + \epsilon) \right) Ev'(h^* + \epsilon) > g' \left( v(h^*) \right) v'(h^*) = U'(h^*).$$

(A.11)

Together with concavity of $U$, this in turn implies that the precautionary premium in Eq. A.10 is positive. Finally, notice that concavity of the function $g$ is equivalent to $I^a > R^a$.

Remark that if $I^a \leq R^a$ then $v''' > 0$ is necessary for a positive precautionary premium and if $v''' \leq 0$ then $I^a > R^a$ is necessary for a positive precautionary premium.

2. We need to show that decreasing absolute risk aversion (DARA) is sufficient for a positive precautionary premium, and it is also necessary under inequality neutrality. To do so, let us define the risk premium $\pi(h^*)$ as follows,

$$Ev(h^* + \epsilon) = v \left( h^* - \pi(h^*) \right).$$

(A.12)

Equivalently, this can be written as

$$H_{A}^{CE} \equiv v^{-1}[Ev(h^* + \epsilon)] = h^* - \pi^*(h^*)$$
Differentiate this identity to find

\[ \frac{E \nu(h^* + \epsilon)}{\nu^{-1}[E \nu(h^* + \epsilon)]} = 1 - \pi'(h^*). \]  (A.13)

It is well known [for example, Gollier (2001, Proposition 4)] that DARA \( \Rightarrow \pi'(h^*) \leq 0 \). It follows that \( \frac{E \nu(h^* + \epsilon)}{\nu^{-1}[E \nu(h^* + \epsilon)]} \geq 1 \). Now we can infer

\[ U'(H^CE_A(c^*_A)) \frac{E \nu'(h^* + \epsilon)}{\nu^{-1}[E \nu(h^* + \epsilon)]} = U'(h^* - \pi^*(h^*)) \frac{E \nu(h^* + \epsilon)}{\nu^{-1}[E \nu(h^* + \epsilon)]} \geq U'(h^* - \pi^*(h^*)) \]  (A.14)

Finally, since \( U \) is concave and \( \pi^*(h^*) \) is positive we have

\[ U'(h^* - \pi^*(h^*)) \geq U'(h^*). \]  (A.15)

This in turn implies

\[ U'(H^CE_A(c^*_A)) \frac{E \nu(h^* + \epsilon)}{\nu^{-1}[E \nu(h^* + \epsilon)]} \geq U'(h^*), \]

which is condition 11 in the text.

Remark that if the social planner is inequality neutral (\( U \) linear) condition (10) in the text becomes

\[ \frac{E \nu(h^* + \epsilon)}{\nu^{-1}[E \nu(h^* + \epsilon)]} \geq 1. \]  (A.16)

As argued above, this condition holds if and only if absolute risk aversion is decreasing.

3. With Rawlsian preferences and, without loss of generality, two individuals, the social objective is to maximize \( \min(v^{-1}(E \nu(h^*_A + \epsilon), h^*_B)). \) The allocation for individual A will be at least as high in the presence of the risk as in the absence of it if the risk decreases the certainty equivalent \( v^{-1}(E \nu(h^*_A + \epsilon)) \). This is true if and only if social preferences display risk aversion.

4. The proof of part 4 of the Proposition (an increase in inequality aversion decreases the optimal investment) can be adapted directly from the proof of Proposition 4 in Kimball and Weil (2009).

\textbf{6.4. Small risk approximation of the precautionary premium under efficiency risk}

We defined the proportional precautionary premium as follows
\[ U' \left( H_A^{CE} (c_A) \right) \frac{\partial H_A^{CE} (c_A)}{\partial c_A} = U' \left( \mu c_A (1 - \hat{\psi}) \right) \mu \]  

(A.17)

where now \( H_A^{CE} (c_A) = v^{-1} \left( Ev(\mu_A c_A) \right) \), with \( \mu_A = \mu \hat{e} \). Following our derivation above of A.2, a second order approximation of \( H_A^{CE} (c_A) \) around \( h^* \) yields

\[ v^{-1} \left( Ev(\mu_A c_A) \right) = h^* + \frac{\nu(r(h^*)}{\nu(h^*)} [h^*]^2 \frac{\sigma^2}{2} + o(\sigma^2) = h^* - R^a [h^*]^2 \frac{\sigma^2}{2} + o(\sigma^2) \]  

(A.18)

Differentiating A.18 with respect to \( c_A \), and for notational clarity suppressing the dependence of \( R^a \) on \( h^* \), we obtain

\[ \frac{\partial H_A^{CE}}{\partial c_A} = \mu \left( 1 - h^* (R^a h^* + 2R^a) \frac{\sigma^2}{2} \right) + o(\sigma^2) \]  

(A.19)

Using A.18, a first order approximation of \( U' \left( H_A^{CE} (c_A) \right) \) around \( h^* \), and again suppressing dependences, yields

\[ U' \left( H_A^{CE} (c_A) \right) = U' - U'' R^a [h^*]^2 \frac{\sigma^2}{2} + o(\sigma^2) \]  

(A.20)

Using A.19 and A.20 we can write the left hand side of A.17 as

\[ \left\{ \mu \left( 1 - h^* (R^a h^* + 2R^a) \frac{\sigma^2}{2} \right) + o(\sigma^2) \right\} \left\{ U' - U'' R^a [h^*]^2 \frac{\sigma^2}{2} + o(\sigma^2) \right\} \]  

(A.21)

Finally, a first order approximation of the right hand side of (A.17) around \( \psi = 0 \) yields

\[ U' \left( h^* (1 - \hat{\psi}) \right) \mu = \mu \left( U' - U'' \hat{\psi} h^* \right) \]  

(A.22)

Setting A.21 equal to A.22, as required by A.17, we obtain

\[ \left\{ \mu \left( 1 - h^* (R^a h^* + 2R^a) \frac{\sigma^2}{2} \right) + o(\sigma^2) \right\} \left\{ U' - U'' R^a [h^*]^2 \frac{\sigma^2}{2} + o(\sigma^2) \right\} = \mu \left( U' - U'' \hat{\psi} h^* \right) \]  

(A.23)

Solving for the premium we obtain

\[ \hat{\psi} = \left[ \frac{U'}{U''} (R^a h^* + 2R^a) + R^a h^* \right] \frac{\sigma^2}{2} + o(\sigma^2) \]  

(A.24)
Keeping in mind the definitions for $I^r$, $\omega$, and $R^r$, Eq. A.24 can equivalently be written as Eq. (12) in the text.

6.5. Proof of Proposition 3

1. The first part of the Proposition says that (a) $I^a > R^a$ and (b) $-\frac{v''(x)}{v''(x)} x \equiv P^r > 2$ are jointly necessary and sufficient for a positive precautionary premium. Let $h^* \equiv \mu c_A$ and $\mu_A = \mu \hat{\epsilon}$, where $\hat{\epsilon}$ is a mean-1 random variable. Equation 14 in the text can be written as

$$g'(Ev(h^*\hat{\epsilon}))Ev'(h^*\hat{\epsilon})\hat{\epsilon} = U'(h^*(1 - \hat{\psi}))$$

where $g(x) \equiv U(v^{-1}(x))$.

Given a mean-1 random variable $x$, Jensen’s inequality implies that $Ev'(h^*x)x \geq v'(h^*)$ if and only if $v'(h^*x)x$ is convex in $x$, a condition which can be written as $h^*[v''''(h^*x)h^*x + 2v''(h^*x)] \geq 0$, or equivalently, $-\frac{v''''(h^*x)h^*x}{v''(h^*x)} \geq 2$.

Risk aversion implies that $Ev(h^*\hat{\epsilon}) < v(h^*)$. If $g$ is concave (which implies $I^a > R^a$) we must then have $g'(Ev(h^*\hat{\epsilon})) > g'(v(h^*))$. Therefore, $-\frac{v''''(h^*x)h^*x}{v''(h^*x)} \geq 2$ together with $I^a > R^a$ imply

$$g'(Ev(h^*\hat{\epsilon}))Ev'(h^*\hat{\epsilon})\hat{\epsilon} > g'(v(h^*))v'(h^*) = U'(h^*).$$

(A.25)

Together with concavity of $U$, this in turn implies that the precautionary premium in Eq. 13 is positive.

2. We will show that decreasing relative risk aversion and $I^r \geq 1$ are sufficient for a positive precautionary premium and that given constant relative risk aversion $I^r \geq 1$ is necessary and sufficient. As shown above, the condition for a positive premium writes as follows,

$$U'(H^*_AC^E) \frac{Ev'(h^*\hat{\epsilon})\hat{\epsilon}}{v'(H^*_AC^E)} \geq U'(h^*)$$

Equivalently, the condition can be written as
The condition \( I^r \geq 1 \) implies that the function \( U'(x)x \) is decreasing in \( x \). Therefore, since \( H_A^{CE} \leq h^* \), we have \( \frac{U'(h^*)h_A^{CE}}{U'(h^*)h^*} \geq 1 \). We will now show that decreasing relative risk aversion implies that \( \frac{Ev(h^*)h_A^{CE}}{V(h^*)h_A^{CE}} \geq 1 \). To do so, let us define the multiplicative risk premium as follows

\[
Ev(h^*) = v\left(h^*(1 - \pi(h^*))\right).
\]

Equivalently, we have

\[
H_A^{CE} \equiv v^{-1}(Ev(h^*)) = h^*(1 - \pi(h^*)).
\]  

Differentiate this function with respect to \( h^* \) to obtain

\[
\frac{Ev(h^*)h^*}{V(h^*)h_A^{CE}} = \left(1 - \pi(h^*)\right) - h^*\pi'(h^*) = \frac{H_A^{CE}}{h^*} - h^*\pi'(h^*),
\]

where the second equality follows from A.27. Dividing both sides by \( H_A^{CE} \) and multiplying both sides by \( h^* \) we obtain,

\[
\frac{Ev(h^*)h^*}{V(h^*)h_A^{CE}} = 1 - h^*\pi'(h^*).
\]

Decreasing relative risk aversion is equivalent to \( \pi'(h^*) < 0 \) (see e.g. Gollier 2001 pg.25), which in turn is equivalent to \( \frac{Ev(h^*)h^*}{V(h^*)h_A^{CE}} \geq 1 \).

Finally, notice that constant relative risk aversion implies that \( \pi'(h^*) = 0 \), so \( \frac{Ev(h^*)h^*}{V(h^*)h_A^{CE}} = 1 \). It follows directly from our previous discussion that \( I^r \geq 1 \) is necessary and sufficient for a positive precautionary premium given the efficiency risk.

3. The proof that with Rawlsian preferences risk aversion is necessary and sufficient for a positive premium is identical to the proof of the third part of Proposition 2.
6.6. Proof under the more general production function

Let $H^*(\epsilon) = H(c_A^*, \theta_A + \epsilon)$. Using the same methods as before, let us re-write condition (15) as

$$g'(Ev(H^*(\epsilon))) Ev'(H^*(\epsilon)) H^*_c(\epsilon) \geq U'(H^*(0)) H^*_c(0) \quad (A.29)$$

By Jensen’s inequality, $Ev'(H^*(\epsilon)) H^*_c(\epsilon) > v'(H^*(0)) H^*_c(0)$ if $v'(H^*(\epsilon)) H^*_c(\epsilon)$ is convex in $\epsilon$ (condition 16 in the text). Furthermore $Ev(H^*(\epsilon)) < v(H^*(0))$ and concavity of $g$ imply that $g'(Ev(H^*(\epsilon))) > g'(v(H^*(0)))$. It follows that given these conditions we have

$$g'(Ev(H^*(\epsilon))) Ev'(H^*(\epsilon)) H^*_c(\epsilon) > g'(v(H^*(0))) v'(H^*(0)) H^*_c(0) = U'(H^*(0)) H^*_c(0). \quad (A.30)$$

References


Courbage, C., Rey, B. (2010) Priority Setting in Health Care and Higher Order Degree Change in Risk. Working paper


