Price Uncertainty, Saving, and Welfare

Diego Nocetti *
Clarkson University

William T. Smith
The University of Memphis

Abstract

We analyze how commodity price uncertainty affects saving behavior and welfare in a dynamic model with multiple commodities, portfolio hedging, and a preference structure that disentangles ordinal preferences, attitudes towards risk, and attitudes towards intertemporal substitution. We show that the effect of price uncertainty on savings boils down to knowing 1) the degree of resistance to intertemporal substitution and 2) the effect that uncertainty has on the certainty-equivalent real interest rate. We also show that, if the certainty-equivalent real interest rate is lower with uncertainty, consumers’ welfare is also lower.

JEL Classification: D91, E21, G11

Keywords: Price uncertainty, Kreps-Porteus preferences, saving, welfare

* We thank two anonymous referees for useful comments and suggestions. Corresponding author: Diego Nocetti, PO Box 5790, Clarkson University, Potsdam, NY. Tel: 315-268-6870. Email: dnocetti@clarkson.edu.
1. **Introduction**

How do consumers respond to uncertainty surrounding commodity prices? It is reasonable to presume that commodity price uncertainty induces some form of precautionary saving.\(^1\) The standard theory of precautionary saving (e.g. Leland, 1968; Sandmo, 1970; Kimball, 1990), which is based on an intertemporal expected utility framework with exogenous income uncertainty and a single good, is unlikely to provide a good answer to the preceding question. First, with price uncertainty the degree of uncertainty is itself endogenous because consumers can substitute consumption inter-temporally and among substitute goods intra-temporally (if relative prices are also changing). Second, in addition to - or instead of - changing the level of savings, consumers may change their portfolio compositions to hedge price volatility if asset returns are correlated with price changes (see, e.g. Kantor 1986). Third, if preferences are defined over multiple goods, then – in a world with uncertainty -- the shape of the utility function governs both risk preferences and ordinal preferences over the goods. It becomes necessary to define multi-commodity measures of risk aversion (see e.g. Kihlstrom and Mirman 1974, 1981; Smith 1999, 2001). Finally, and not unique to the case of price uncertainty, the standard theory of precautionary saving does not distinguish between preferences towards risk and preferences towards intertemporal substitution, which is of critical importance given the dynamic aspect of saving and the opportunity for intertemporal substitution to which we alluded before (Kimball and Weil, 2009).

The objective of this paper is to analyze precisely how uncertainty surrounding (the growth rate of) commodity prices affects consumption and saving behavior. Eaton (1980)

\(^1\) We will generally refer to the stochastic nature of the growth rate of commodity prices as price uncertainty.
analyzed this question in a two-period and two goods framework with intertemporal expected utility preferences. We develop and solve in closed form a model with four features:

- An infinite horizon with N goods
- Financial assets correlated with the growth rate of commodity prices.
- A preference specification that disentangles risk aversion from preferences towards the goods in absence of risk.
- A class of recursive preferences developed by Kreps and Porteus (1978), appropriately adapted to the case of multiple commodities, that disentangles risk aversion and intertemporal substitution preferences.

We show that the effect of price uncertainty on savings boils down to knowing 1) the size of the elasticity of intertemporal substitution and 2) the effect that price uncertainty has on the certainty-equivalent real interest rate. This latter effect, in turn, depends on the consumer’s degree of risk aversion, on the opportunity for intra-temporal substitution among the goods (e.g. the degree to which prices of different goods co-vary), and on the degree to which the inflation rate co-varies with stock-market returns.

Besides the work of Eaton (1980), surprisingly little work has been done on the relation between saving and price uncertainty. Our results, however, are also tightly related to the effect of price uncertainty on consumer’s welfare, which is a topic that has received significant attention (e.g. Waugh, 1944; Eaton, 1980; Turnovsky et al., 1980; Baye, 1985; Ni and Raymon, 2004). We show that price uncertainty decreases welfare if it also decreases the certainty-equivalent real interest rate. This analysis sheds light into some of the counter-intuitive results

---

recently provided by Ni and Raymon (2004) in an inter-temporal framework with Kreps-Porteus preferences.

The rest of the paper proceeds as follows. In Section 2 we present the model and we discuss in some depth the extension of Kreps-Porteus preferences to the case of multiple commodities. In Sections 3 and 4 we establish our main results. In Section 5 we present numerical examples of the strength of precautionary saving and of the welfare loss/gain from eliminating price volatility. Section 6 concludes.

2. The Model

2.1. Prices, Returns, and the Budget Constraint

Imagine a consumer who lives forever. In each period \( t \in (0, \infty) \) he consumes \( N \) goods, \( x_{j,t}, j = 1, \ldots, N \). The price of good \( j \) in period \( t \) is \( p_{j,t} \). Denote the column vectors of quantities and prices in period \( t \) by \( \mathbf{x}_t = [x_{1,t}, \ldots, x_{N,t}] \) and \( \mathbf{p}_t = [p_{1,t}, \ldots, p_{j,t}] \) respectively. Over a short interval \( \Delta t \), the prices of the goods follow a vector diffusion

\[
\Delta p_{j,t}/p_{j,t} = (p_{j,t+\Delta t} - p_{j,t})/p_{j,t} = \pi_j \Delta t + \sigma_j \Delta z_{j,p,t} \quad j = 1, \ldots, N. \tag{1}
\]

\( \pi_j \) and \( \sigma_j \) are, respectively, the expected growth rate and standard deviation of the growth rate of good \( j \)'s price, and \( \Delta z_{j,p,t} \) is a standard-normal innovation that will in the limit become a Wiener process. We assume that the means and standard deviations of the growth rates are constant, so that in the continuous-time limit the process will become a vector geometric Brownian motion.

We also assume that all prices are expected to grow at the same rate, so that \( \pi_j = \pi, \forall j \).

Furthermore, we will denote the correlation coefficient of the innovations as \( \varphi_{j,t} \).

**Remark.** We refer to an economy with \( \sigma_j > 0 \) for at least some \( j \) as an economy with price uncertainty (or price volatility).
In addition, there are two assets. One is a bond that provides a certain nominal rate of return $r$. The price of the other asset $q_t$ follows a diffusion process

$$\Delta q_t / q_t = (q_{t+\Delta t} - q_t) / q_t = \omega_q \Delta t + \sigma_q \Delta z_{q,t}. \quad (2)$$

where $\omega_q$ and $\sigma_q$ are, respectively, the expected nominal return and the standard deviation of the return, and $\Delta z_{q,t}$ is a standard-normal innovation that will in the limit become a Wiener process.

As with prices we assume that the mean and standard deviation are constant, so that Equation (2) turns into a geometric Brownian motion in the continuous time limit. We set $\omega_q = r + \mu$, so $\mu$ is the expected excess return of this asset. The innovation of this asset is correlated with the shocks to the rate of change of good $j$’s price, with a coefficient of correlation $\phi_{j,q}$.

The consumer’s wealth at time $t$ is $w_t$, of which he invests a share $\lambda_t$ in the risky asset and a share $(1 - \lambda_t)$ in the nominally riskless asset. The flow budget constraint over the interval $\Delta t$ is then

$$\Delta w_t = w_{t+\Delta t} - w_t = [(r + \mu \lambda_t)w_t - C_t] \Delta t + w_t \lambda_t \sigma_q \Delta z_{q,t}. \quad (3)$$

where $C_t = p_t^j x_t$ is total expenditures in the $N$ goods, or simply, consumption.

### 2.2. Preferences

The canonical way of formulating a problem like this in continuous time would be to assert time-separable preferences defined over consumptions in each period:

$$E \int_0^\infty e^{-\delta t} u(x_t) \, dt, \quad (4)$$

where $\delta$ is the constant rate of time preference. It is well known that if $x_t$ were a singleton, rather than a vector, then the felicity function $u(x_t)$ would confound risk aversion and willingness to

---

3 Although we refer to the bond as a riskless asset one should note that, because the inflation rate is stochastic, the real return on the bond is also risky.
substitute over time. Similarly, if $x_t$ were a vector, then the utility function would confound risk aversion and ordinal preferences between the different goods in the same time period. In this general setting – with multiple goods, many periods of time, and uncertainty – these problems are compounded. The curvature of the felicity function now governs three distinct aspects of preferences, risk aversion, preferences for intertemporal substitution, and ordinal preferences across the different goods.

We want to evaluate the effect of price uncertainty in our infinite horizon model with a utility functional that disentangles these different dimensions of preferences. We achieve this by merging results from two literatures, one on recursive preferences in dynamic models, the other on multivariate risk aversion in static models. We will briefly discuss these literatures separately, and then put them together to arrive at our preference functional.

2.2.1 Risk Aversion and Intertemporal Substitution

To separate risk preferences from preferences for intertemporal substitution in a dynamic setting we adopt the class of recursive preferences developed by Kreps and Porteus (1978), which is a multi-period extension of a two-period model of Selden (1978, 1979). Because of its tractability, we use the special case of the Generalized Isoelastic (GIE) utility function, advocated by Svensson (1989), Weil (1990), and Epstein and Zin (1991). Under GIE preferences with a single good $c_t$ utility obeys the recursion

$$(1 - \gamma)u(t) = \lim_{\Delta t \to 0} \left\{c_t^{1 - \frac{1}{\varepsilon}} \Delta t + e^{-\delta \Delta t} \left[(1 - \gamma)Eu(t + \Delta t)\right]^{1 - \frac{1}{\varepsilon}}\right\}^{\frac{1 - \frac{1}{\gamma}}{1 - \frac{1}{\varepsilon}}}.$$  \hspace{1em} (5)

The expression $[(1 - \gamma)Eu(t + \Delta t)]^{\frac{1}{1 - \gamma}}$ is certainty-equivalent future utility. On inspection it follows that current utility is a constant-elasticity-of-substitution (CES) function of current felicity and certainty-equivalent future utility. This means that we can interpret $\gamma$ as the
coefficient of relative risk aversion over timeless gambles, while we can interpret $\epsilon$ as the elasticity of intertemporal substitution (EIS) defined over riskless consumption paths. When $\gamma = \frac{1}{\epsilon}$, Equation (5) reduces to the standard specification of isoelastic preferences.

2.2.2 Risk Aversion and Ordinal Preferences over Commodities

To separate risk aversion from ordinal preferences over the different goods we apply the analysis of multivariate risk aversion developed by Kihlstrom and Mirman [(1974, 1981)]. Consider a simple utility function defined over two goods, $u(x_1, x_2)$. What does risk aversion mean in this setting? Kihlstrom and Mirman argued as follows. First, we know from Pratt (1964) that one person is more risk averse than another if and only if his utility function is more concave than the other’s. Second, Debreu (1976) proved that any concave utility function possesses a “least concave” representation. Finally, Kihlstrom and Mirman proved that if the utility function is homothetic, then the least concave representation is linearly homogeneous. In other words, if $u(x_1, x_2)$ is homothetic, then it can be expressed as

$$u(x_1, x_2) = v[f(x_1, x_2)],$$

where $v(\cdot)$ is concave and $f(\cdot)$ is homogeneous of degree one. The concavity of $v(\cdot)$ implies – à la Pratt – that $v[f(x_1, x_2)]$ is more risk averse than $f(x_1, x_2)$.

What all this means in practice is that we can interpret the curvature of $v(\cdot)$ as governing risk aversion proper and the curvature of $f(\cdot)$ as governing preferences over $x_1$ and $x_2$. An illuminating example is the Cobb-Douglas utility function

$$u(x_1, x_2) = \frac{(x_1^{a_1} x_2^{a_2})^{1-\gamma}}{1-\gamma}. \quad (6)$$

Notice that this can be rewritten as
The “aggregator” function inside the parenthesis is now linearly homogeneous, so its exponents are weights governing preferences over the two goods. The transforming function is \((\cdot)^{(1-\gamma)(a_1+a_2)}\), so that what Smith (2001) calls the “effective” coefficient of relative risk aversion is \(R = 1 - (1 - \gamma)(a_1 + a_2)\).

### 2.2.3 Risk Aversion, Intertemporal Substitution, and Ordinal Preferences

Since we need to disentangle all three aspects of preferences we write our utility functional as

\[
(1 - \gamma)u(t) = \lim_{\Delta t \to 0} \left\{ \left( \prod_{j=1}^{N} x_{j,t}^{a_j} \right)^{1 - \gamma} \Delta t + e^{-\delta \Delta t} [(1 - \gamma)Eu(t + \Delta t)]^{1 - \gamma} \right\}^{\frac{1}{1-\gamma}}. \tag{8}
\]

The effective measure of relative risk aversion is now \(R = 1 - (1 - \gamma) \sum_{j=1}^{N} a_i\). Similarly, the effective measure of the EIS is \(\theta = 1/\left[1 - \left(1 - \frac{1}{\epsilon_j}\right) \sum_{j=1}^{N} a_i \right]\). To avoid notational clutter we will assume that \(\sum_{j=1}^{N} a_j = 1\), so \(\gamma\) and \(\epsilon\) effectively measure the degree of relative risk aversion and the EIS, respectively.

### 3. Price uncertainty and Consumption

Before establishing optimal consumption it will prove convenient to define a price index.

Given our Cobb-Douglas aggregator, the price index \(P_t\) takes the form

\[
P_t = \prod_{j=1}^{N} p_{j,t}^{a_j}. \tag{9}
\]

Given this definition, one can show using Ito’s lemma that the expected growth rate of the price index, i.e. the expected inflation rate, is

\[
\Pi = \pi - \sum_{j=1}^{N} a_j \frac{\sigma_j^2}{2} + \frac{\sigma^2}{2}. \tag{10}
\]
where \( \sigma_p^2 = \sum_{j=1}^{N} \sum_{i=1}^{N} a_j a_i \varphi_{j,i} \sigma_j \sigma_i \) is the variance of the inflation rate. Furthermore, the correlation between the asset return and the inflation rate is

\[
\varphi_{p,q} = \frac{\sum_{j=1}^{N} a_j \varphi_{j,q} \sigma_j}{\sigma_p}.
\]

For future reference, we note that \( \Pi \leq \pi \): the expected inflation rate is lower than the growth rate of the individual prices growth rate. The special case in which \( \Pi = \pi \) occurs when \( \varphi_{j,i} = 1 \) and \( \sigma_j = \sigma_i \) for all \( j \) and \( i \), that is, there are proportional price changes across goods. This is equivalent to a situation with a single good. We will first analyze this special case, which we will refer as “pure inflation uncertainty.”

3.1. Pure Inflation Uncertainty

Consider first a situation in which shocks are proportional to all goods and let us disregard the portfolio choice for the moment. Given these assumptions, in the Appendix we show

**PROPOSITION 1.** Suppose that there is pure inflation uncertainty and that \( \lambda_t = 0 \). Then, in the continuous-time limit, consumption equals

\[
C_t = \{\varepsilon \delta + (1 - \varepsilon) \hat{r}^{CE}\} w_t
\]

where \( \hat{r}^{CE} = \hat{r} - \gamma (\hat{\sigma}_p^2 / 2) \) is the certainty-equivalent real rate of return, with \( \hat{r} = r - \pi + \hat{\sigma}_p^2 \), the expected real return from the nominal bond, and \( \hat{\sigma}_p^2 = \sigma_j^2 \forall j \).

---

4 In this section we assume that \( \lambda_t = 0 \) for ease of exposition. In essence, we are assuming that the consumer does not have access to equity markets and can only invest in the nominally riskless bond. In the next section we analyze the more general case.

5 The presence of \( \hat{\sigma}_p^2 \) in the real rate of return (the “expected return” effect) arises because real wealth is convex in the price level. A simple application of Ito’s lemma then shows that the expected depreciation rate of a unit of money equals \( Ed(1/P)/(1/P) = -\pi + \sigma_p^2 \) (e.g., Fischer, 1975).
As in the case of interest rate uncertainty analyzed by Weil (1990), evaluating the effect of pure inflation uncertainty on consumption boils down to analyzing how uncertainty affects the certainty-equivalent real interest rate. Specifically, inflation uncertainty has two distinct effects:

- First, it increases the riskiness of the real interest rate (decreasing $\hat{r}^{CE}$).
- Second, it increases the expected real interest rate (increasing $\hat{r}^{CE}$).

The distinction between risk and intertemporal smoothing preferences is central to our understanding of how these effects change consumption and saving. As shown by Selden (1979) and by Weil (1990) capital risk increases saving and decreases consumption if the elasticity of intertemporal substitution is smaller than one. The degree of risk aversion only affects the magnitude of this effect. The other effect, an increase in the expected interest rate, goes in the opposite direction, increasing saving and decreasing consumption if the elasticity of intertemporal substitution is larger than one. Risk aversion plays no role whatsoever.

To evaluate precisely the effect of pure inflation uncertainty on consumption and savings note that we can re-write Eq. (11) as

$$C_t = \left\{ \varepsilon \delta + (1 - \varepsilon) \left\{ r - \pi + (2 - \gamma) \frac{\sigma^2}{2} \right\} \right\} w_t. \tag{11}'$$

This has the obvious corollary,

**COROLLARY 1.** Assume that $\lambda_t = 0$. Pure inflation uncertainty decreases or increases $\hat{r}^{CE}$ according to $\gamma \gtrless 2$. It decreases or increases consumption according to $(1 - \varepsilon)(2 - \gamma) \leq 0$.

In other words, if the capital risk effect is stronger than the expected return effect, which occurs when $\gamma > 2$, the consumer faces a lower effective interest rate, increasing saving if $\varepsilon < 1$.

### 3.2. General Setting

We now consider the more general setting with multiple sources of risk that may not be perfectly correlated and the possibility of portfolio hedging. The following proposition, which
we prove in the Appendix, establishes optimal consumption and portfolio choice in the more
general case

**PROPOSITION 2.** In the continuous-time limit, the demand for the risky asset is

\[
\lambda^* = \frac{\mu}{\gamma \sigma_q} + (\gamma - 1) \frac{\phi_{p,q} \sigma_p}{\gamma \sigma_q}
\]  

(12)

and consumption equals

\[
C_t = \{\varepsilon \delta + (1 - \varepsilon)r^{CE}\} w_t
\]

(13)

where \( r^{CE} = \bar{r} - \gamma(\bar{\sigma}^2/2) \) is the certainty-equivalent real rate of return from the portfolio, with

\[
\bar{r} = r + (\mu - \varphi_{p,q} \sigma_p \sigma_q) \lambda^* - \Pi + \sigma_p^2, \text{ the expected real return from the portfolio, and } \bar{\sigma}^2 = \sigma_q^2(\lambda^*)^2 + \sigma_p^2 - 2\lambda^* \varphi_{p,q} \sigma_p \sigma_q, \text{ the variance of the real return.}
\]

The first term in Equation (12) represents the standard portfolio demand policy [e.g. Merton 1969]. In addition to this, consumers that are sufficiently risk averse (\( \gamma > 1 \)) increase (decrease) the demand for the risky asset if \( \varphi_{p,q} > 0 \) (if \( \varphi_{p,q} < 0 \)) as a hedging mechanism. That is, by investing in the asset, the consumer expects to earn a relatively high return when the inflation rate is high. This is consistent with the evidence presented by Kantor (1986) and by Chen et al (1986).\(^6\) The hedging demand increases, in absolute terms, with the degree of relative risk aversion and with the correlation coefficient \( \varphi_{p,q} \) (since in this case the asset is a more effective hedging tool). As shown by Svensson (1989) and by Weil (1989), the optimal portfolio policy

---

\(^6\) Kantor (1986) showed that consumers decreased the demand of stock market assets and increased the demand of short term government bonds in response to the increase in inflation uncertainty in the 1970s. Consistent with our analysis, he argued that, given the empirical regularity that stock market returns have a negative correlation with the inflation rate (e.g. Bodie, 1976; Fama and Schwert, 1977), this shift occurred due to hedging motives. Chen et al (1986) showed that assets that have a positive correlation with the inflation rate carry a lower risk premium, which is consistent with the fact that the demand for those assets is higher.
does not depend on intertemporal smoothing preferences. This is still true with the additional hedging demand.

Now consider the optimal consumption rule in Eq. (13). As before, consumption increases with the rate of time preference and it increases (decreases) with the certainty-equivalent real interest rate if the elasticity of intertemporal substitution is smaller (larger) than one. Uncertainty surrounding the growth rate of commodity prices affects the certainty-equivalent real interest rate in three different ways:

- First, it increases the riskiness of the real return of the portfolio (decreasing $r^{CE}$).
- Second, it increases the expected real interest rate (increasing $r^{CE}$), both directly, by making the inflation rate uncertain, and indirectly, by decreasing the expected inflation rate.
- Third, there is a portfolio hedging effect. Given a positive portfolio share $\lambda^*$, if the inflation rate is positively correlated with equity returns, the expected real return of the portfolio and the volatility of the portfolio are lower, decreasing and increasing, respectively, $r^{CE}$. The opposite is true if the inflation rate is negatively correlated with equity returns.

We can provide further intuition by rewriting Eq. (13) as

$$C_t = \left\{ \epsilon \delta + (1 - \epsilon) \left\{ r - \pi + \frac{\mu^2}{\gamma \sigma^2_q} + \Omega \right\} \right\} w_t \quad \text{(13)'}$$

where,

$$\Omega = \sum_{j=1}^N a_j \sigma_j^2 + (1 - \gamma) \left[ 1 + \left( \varphi_{P,q} \right)^2 \frac{(1-\gamma)}{\gamma} \right] \frac{\sigma^2_p}{2}. \quad \text{(14)}$$

Clearly, the term $\Omega$, which combines the different effects mentioned above, determines the overall effect of price uncertainty on the certainty-equivalent real interest rate. We can therefore conclude
COROLLARY 2. If $\Omega < 0 \ (\Omega > 0)$ then $r^{CE}$ is lower (higher) under uncertainty, decreasing consumption and increasing savings if $\varepsilon < 1 \ (\varepsilon > 1)$.

A number of special cases are of particular interest. Consider first the case in which there is no hedging demand (i.e. $\varphi_{p,q} = 0$). Then, from Eq. (14) we obtain

COROLLARY 3. When asset returns are uncorrelated with the inflation rate, $\Omega \leq 0$ according to $\gamma \geq 1 + \frac{\sum_{j=1}^{N} a_j \sigma_j^2}{\sigma_p^2}$.

Comparing Corollaries 1 and 3 one can see that pure inflation uncertainty is more likely to decrease $r^{CE}$ than the case in which relative prices also change (i.e. $\sum_{j=1}^{N} a_j \sigma_j^2 / \sigma_p^2 \geq 1$).

Intuitively, when relative prices are expected to change, the consumer can reduce the negative effect of inflation by substituting among the goods. As a result, the expected inflation rate is lower and $r^{CE}$ is higher. If $\varepsilon < 1 \ (\varepsilon > 1)$, then savings will be lower (higher) when shocks do not have a perfect positive correlation. In fact, if only relative prices change and the inflation rate is constant, $r^{CE}$ is unambiguously higher under uncertainty, increasing savings only if $\varepsilon > 1$.

More generally, when the consumer can reduce his negative exposure to inflation uncertainty by using financial assets the certainty-equivalent real interest rate is higher. A larger degree of correlation (in absolute terms) implies that the asset works better as a hedging tool, so the certainty-equivalent real interest rate increases, reducing consumption and increasing savings if $\varepsilon > 1$. In the extreme case of perfect correlation between the asset return and the inflation rate we obtain

COROLLARY 4. When asset returns are perfectly correlated with the inflation rate, $\Omega > 0$.

Uncertainty decreases consumption and increases savings only if $\varepsilon > 1$.

Given perfect correlation the certainty-equivalent real interest rate is unambiguously higher under uncertainty. Therefore, whether consumption increases or decreases with price variability
depends exclusively on attitudes towards intertemporal smoothing. Individuals that are more resistant to intertemporal substitution consume (save) more (less). The degree of risk aversion only changes the magnitude of consumption and savings.

Finally, under pure inflation uncertainty but with portfolio hedging we can generalize Corollary 1 as follows

**COROLLARY 5.** Under pure inflation uncertainty and a correlation between the inflation rate and asset returns equal to \( \varphi \), \( \Omega \leq 0 \) according \( \gamma \geq 1 + \frac{1}{\sqrt{1 - (\varphi)^2}} \).

Clearly, the larger the correlation, the more likely it is that \( r^{CE} \) increases with inflation uncertainty, decreasing consumption and increasing savings if \( \varepsilon > 1 \). If \( |\varphi| = 1 \), then \( r^{CE} \) is unambiguously higher under uncertainty, as stated in Corollary 4.

4. **Price Uncertainty and Welfare**

To evaluate the effect of price uncertainty on welfare we follow the method proposed by Epaulard and Pommeret (2003). Denote the value function at time 0 as a function of real wealth, \( \frac{w_0}{P_0} \), and of the certainty-equivalent interest rate in the presence of price uncertainty, \( r^{CE} \), by \( u \left( \frac{w_0}{P_0}, r^{CE} \right) \). The welfare cost of price uncertainty is defined as the percentage \( k \) of current real wealth that the consumer is willing to give up to live in an economy with certain commodity prices. This cost is the solution to the equation

\[
u \left( \frac{w_0}{P_0}, r^{CE} \right) = u \left( (1 - k) \frac{w_0}{P_0}, \tilde{r}^{CE} \right), \tag{15}\]

where \( \tilde{r}^{CE} = r - \pi + \frac{\mu^2}{\gamma \sigma_q} \) is the certainty equivalent real interest rate when all prices grow at a constant rate \( \pi \). In the Appendix we show
PROPOSITION 3. The welfare cost of price uncertainty is given by

\[
k = 1 - \left[ \frac{\varepsilon \delta + (1-\varepsilon) r E}{\varepsilon \delta + (1-\varepsilon) r E} \right]^{\frac{1}{1-\varepsilon}} = 1 - \left[ \frac{\varepsilon \delta + (1-\varepsilon) \left( r - \pi + \frac{\mu^2}{\gamma \sigma^2} \right)}{\varepsilon \delta + (1-\varepsilon) \left( r - \pi + \frac{\mu^2}{\gamma \sigma^2} \right)} \right]^{\frac{1}{1-\varepsilon}}.
\] (16)

If \( \Omega < 0 \) \( (\Omega > 0) \), welfare is higher (lower) when all prices grow at a constant rate.

In other words, if the certainty-equivalent real interest rate is higher when all prices grow at a constant rate welfare is also higher in this case. Therefore, our previous analysis implies that price uncertainty is more likely to reduce welfare if 1) the consumer is highly averse to risk, 2) there is limited intra-temporal substitution among the goods (e.g. proportional changes in prices, those goods which the consumer spends a large share of her income have more variable prices, and 3) asset returns are a poor hedge against inflation variability. If only relative prices change but the aggregate level of prices is constant, or if there is an asset with returns that are perfectly correlated with the inflation rate, then, price uncertainty unambiguously increases consumer’s welfare.

These results help elucidate some of the conclusions presented by Ni and Raymon (2004). They evaluated the effect of mean preserving spreads in the distribution of one or more future prices in a one-good setting with Kreps-Porteus preferences. They argued that 1) price volatility is more likely to affect welfare negatively in an inter-temporal setting as compared to a static setting and 2) price volatility is more likely to reduce welfare if it occurs later in life (e.g. the last period of life). Taken together, these two results are counterintuitive. One would expect that, if price volatility is more harmful (less beneficial) when it occurs later in life, then, welfare should also be lower in a static setting. This is, in fact, the case. The counterintuitive result in Ni and Raymon’s paper arises from their first conclusion, which they reached by comparing relative price volatility in a static setting with price volatility in a two-period one-good setting (or
equivalently, with proportional changes in all prices). Instead, we showed that it is the opportunity for intra-temporal substitution that arises when shocks are not proportional to all goods that diminishes the negative effects of price (level or growth) volatility.

Given this, it is also simple to understand Ni and Raymon's second conclusion: volatility of the relative price of consumption over time creates an opportunity for inter-temporal substitution across periods that does not exist in a static setting. This opportunity does not exist either if 1) prices are uncertain only in the last period of life or 2) when shocks are permanent, as in our model where prices follow a vector geometric Brownian motion.

5. **Calibration**

To calibrate the model we need time series data of commodity prices and equity returns and estimates of relative risk aversion and the EIS. We use annual price series (1947-2009) for personal consumption expenditures by major type of product (16 types) provided by the Bureau of Economic Analysis (NIPA table 2.3.7). The weight $a_j$ given to each type of product is the amount spent in the product as a percentage of total spending in the year 2005. For the same period (1947-2009) we calculate first and second moments of the annual return of the S&P 500 and its correlation with each major type of product. We also set $r = \pi = 3\%$ and $\delta = 0.02$. Table 1 summarizes our calculations.

Empirical estimates of relative risk aversion and the EIS vary widely. For example, some estimates of $\gamma$ range from 1 to 4 (Friend and Blume, 1975; Hansen and Singleton, 1983; Epstein and Zin, 1991). Mehra and Prescott (1985) think that $\gamma = 10$ is the upper limit of plausible values, but Kandel and Stambaugh (1991) make a case for risk aversion as large as $\gamma = 29$. Most

---

7 In an appendix, available upon request, we provide all the data and the calculations necessary to perform the calibration.
empirical estimates of $\varepsilon$ place it far below one (e.g. Epstein and Zin, 1991; Attanasio and Weber 1993, Atkenson and Ogaki 1996) and sometimes close to zero (e.g. Hall, 1988). However, there is also some evidence that $\varepsilon$ may be greater than one (Attanasio and Weber, 1989; Buffman and Leiderman, 1990). Given this wide range of estimates we will present numerical examples for different values of $\gamma$ and $\varepsilon$.

Tables 2 shows the percentage decrease in consumption due to the presence of uncertainty, $[1 - C_t(\bar{r}^{CE}) / C_t(\bar{r}^{CE})]$, i.e. the strength of the precautionary saving motive. Table 3 shows the welfare cost of price uncertainty, $k$ as defined in Eq. (16).

The calibration shows a number of important features. First, the threshold value of relative risk aversion that makes the consumer indifferent about the presence and absence of uncertainty is 3.27. As explained above, this is larger than the threshold with pure inflation uncertainty ($\gamma = 2$) for two reasons: 1) the possibility to substitute goods intra-temporally due to changes in relative prices and 2) the opportunity to hedge inflation risk with financial assets. The main difference arises from intra-temporal substitutability since $1 + \frac{\sum_{j=1}^{N} a_j \sigma_j^2}{\sigma_p^2} = 3.17$. This confirms Reis and Watson’s (2010) finding that relative price volatility is an important component of aggregate price volatility.

Second, the magnitude of the precautionary saving motive and of the welfare loss from living in a world with uncertain commodity prices depend to a great extent on the estimates of $\gamma$ and $\varepsilon$ that one considers more accurate. For low to moderate values of risk aversion, say 1-6, the effect of uncertainty is quite small. This should not be surprising. It is well known that, for moderate levels of risk aversion, the gains from (consumption) stabilization in the United States...
are extremely small (e.g. Lucas 2003). Given the observed low volatility of prices, the same is true in our framework.

For large values of risk aversion, however, the reduction in saving and the gain from stabilization could be substantial (e.g. for risk aversion equal to 10 and an EIS equal to 0.1 the representative consumer would be willing to give up about 15% of his current wealth to face stable prices). The separation of preferences towards risk and towards intertemporal substitution is also important. Suppose, for example, that we consider Hall’s (1988) estimate of the EIS (about 0.05) to be accurate. In an intertemporal expected utility model we would then constrain relative risk aversion to be 20. The consumer would then be willing to give up about 70% of his or her current wealth to eliminate price volatility. If, however, we take a more likely value of relative risk aversion, say 6, then he or she would be willing to give up about 4% of his or her current wealth. In other words, an intertemporal expected utility model would grossly overstate the gains from stabilization.

In summary, our calculations show that, for more plausible values of risk aversion, the effect of price volatility on consumption and welfare in the US is quite small. Of course, this may not be the case in countries where price volatility is higher. For example, for European OECD countries, the average standard deviation of inflation for the period 1971-2009 was about 0.0336 while the average inflation rate was about 4.54%. Let us then assume that a) \( r = \pi = 4.54\% \), b) the parameters \( \mu, \sigma_q, \varphi_{p,q} \) are the same as in the US (see table 1), and c) there is a single good, so 0.0336 reflects the standard deviation in the data and in the model. Then, given a degree of risk aversion of 6, the elimination of inflation volatility would increase consumption by about 12% and the consumer would be willing to give up a similar percentage in terms of current wealth to live in the certain economy.
6. Conclusion

We studied how uncertainty surrounding the growth rate of commodity prices affects saving behavior and welfare in a dynamic model with a utility functional that separates ordinal preferences, risk aversion, and intertemporal substitution preferences. Price uncertainty induces precautionary saving if the EIS is lower (higher) than one and the certainty-equivalent real interest rate decreases (increases) with such uncertainty. If uncertainty decreases the certainty equivalent rate, welfare also decreases. This is more likely to occur for highly risk averse consumers, if there is limited intra-temporal substitution among the goods, and if asset returns are a poor hedge against inflation.

Our calibration with US data shows that price uncertainty induces precautionary saving and reduces welfare for those consumers with a coefficient of relative risk aversion larger than about 3 (given an EIS lower than 1). Given that most likely estimates of risk aversion are not much different from 3 and given the fairly stable growth rate of prices in the US it is unlikely that price volatility has a large impact (positive or negative) empirically. Our example with data from European countries suggests, however, that in countries with more volatile prices the effect could be substantial. Future research could improve our calibration by 1) considering a more realistic inflation process (e.g. one that captures the serial correlation), 2) Evaluating the effect of shocks that are not permanent (e.g. one period) and 3) considering a more general preference structure (e.g. one that accounts for the fact that commodities with more volatile prices are also more inelastic - food and gasoline).  

8 In particular, the assumption that the intra-temporal utility is Cobb-Douglas, which implies that the elasticity of intra-temporal substitution equals one, makes the model tractable. However, a more complete calibration could include different and possibly non-constant values of the elasticity of substitution.
Appendix

Since Proposition 2 is more general than Proposition 1 we will start by proving the former.

Proof of Proposition 2.

The value function satisfies the following recursion:

$$(1 - \gamma) u(w_t, p_t) = \lim_{\Delta t \to 0} \max_{(x_{j,t})^N_{j=1}} \left\{ \left[ \prod_{j=1}^{N} \frac{a_j}{\varepsilon} \right]^{\frac{1}{\varepsilon}} \Delta t + e^{-\delta \Delta t} \left[ (1 - \gamma) E[u(w_{t+\Delta t}, p_{t+\Delta t})] \right]^{\frac{1-\gamma}{1-\varepsilon}} \right\}. \quad (A.1)$$

Conjecture that

$$u(w_t, p_t) = \left( A^{1-\gamma} w^{1-\gamma}_t \prod_{j=1}^{N} p^{1-\gamma}_j \right) (1 - \gamma)^{-1}, \quad (A.2)$$

$$x_{j,t} = \eta_j \left( w_t / p_{j,t} \right), \quad (A.3)$$

where $A$ and $\eta_j$ are $N+1$ constants to be determined.

Using these conjectures, the expression in braces on the right-hand side of Equation (A.1) becomes

$$\left( \prod_{j=1}^{N} a_j / \varepsilon \right)^{\frac{1}{\varepsilon}} w_t^{1-\rho} \prod_{j=1}^{N} p^{1-\gamma}_j \Delta t + e^{-\delta \Delta t} A^{1-\gamma} \left[ E w_{t+\Delta t}^{1-\gamma} \prod_{j=1}^{N} p^{1-\gamma}_j \right]^{\frac{1}{1-\varepsilon}}. \quad (A.4)$$

Using Equation (A.3) into the expectation in Equation (A.4), together with the processes of prices and asset returns in Equations (1) and (2) and the flow budget constraint in Equation (3), and taking a Taylor-series expansion around $\Delta t = 0$ one obtains

$$E w_{t+\Delta t}^{1-\gamma} \prod_{j=1}^{N} p^{1-\gamma}_j = w_t^{1-\gamma} \prod_{j=1}^{N} p^{1-\gamma}_j \left\{ 1 + \left[ (1 - \gamma) \left( r - \pi + \mu_q \lambda_t - \sum_{j=1}^{N} \eta_j \right) + (1 - \gamma) \sum_{i=1}^{N} a_i (\sigma_i^2 / 2) + (1 - \gamma)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j (\sigma_{i,j} / 2) - \gamma (1 - \gamma) (\lambda_t)^2 (\sigma_q^2 / 2) - (1 - \gamma)^2 \lambda_t \sum_{j=1}^{N} \sum_{i=1}^{N} a_i \sigma_{i,j} \right] \Delta t \right\}. \quad (A.5)$$

One can simplify the notation by noting that Equation (A.2) can also be written as
\[ u(w_t, p_t) = \left( A^{1-\gamma} w_t^{(1-\gamma)} p_t^{-(1-\gamma)} \right) (1 - \gamma)^{-1}, \]  

(A.2)'

where \( P_t = \prod_{j=1}^{N} p_{j,t}^{a_j} \) is the aggregate price level. Using Ito’s lemma we have that the expected inflation rate and the variance of the inflation rate are, as argued in the text,

\[ \Pi = \pi \sum_{j=1}^{N} a_j - \sum_{i=1}^{N} a_j \left( \sigma_i^2 / 2 \right) + \sigma_P^2 \]

\[ \sigma_P^2 = \sum_{j=1}^{N} \sum_{i=1}^{N} a_j a_i \varphi_{j,i} \sigma_i \sigma_j. \]

Also, the correlation between the asset return and the inflation rate is

\[ \varphi_{P,q} = \frac{\sum_{j=1}^{N} a_j \varphi_{jq} \sigma_j}{\sigma_P}. \]

We can now rewrite Equation (A.5) as

\[ E w_t^{(1-\gamma)} p_t^{-(1-\gamma)} = E w_t^{(1-\gamma)} p_t^{-(1-\gamma)} \{ 1 + (1 - \gamma)(r - \Pi + (2 - \gamma) \sigma_P^2 + \mu_q \lambda_t - \sum_{j=1}^{N} \eta_j) - \gamma(1 - \gamma)(\lambda_t)^2 (\sigma_q^2 / 2) - (1 - \gamma)^2 \lambda_t \varphi_{P,q} \sigma_P \sigma_q \delta t \} \]

(A.5)'

Plugging Equations (A.4) and (A.5)' back into Equation (A.1), expanding the resulting expression with a Taylor series, dividing by \( \Delta t \) and taking the limit as \( \Delta t \to 0 \) yields

\[ 0 = \max_{\{ \chi_{j,t} \}_{j=1}^{N}, \lambda_t} \left( \prod_{j=1}^{N} a_j / A \right)^{1-\gamma} - \delta + \left( 1 - \frac{1}{\gamma} \right) (r - \Pi + (2 - \gamma) \sigma_P^2 + \mu_q \lambda_t - \sum_{j=1}^{N} \eta_j) + \gamma \left( 1 - \frac{1}{\gamma} \right) (\lambda_t)^2 (\sigma_q^2 / 2) - \left( 1 - \frac{1}{\gamma} \right) (1 - \gamma) \lambda_t \varphi_{P,q} \sigma_P \sigma_q \]

(A.6)

Now perform the indicated maximization to arrive at the N+1 first-order conditions

\[ (a_i / \eta_i) \left( \prod_{j=1}^{N} a_j / A \right)^{1-\gamma} = 1 \quad \text{for } i = 1, \ldots, N \quad \text{(A.7)} \]

\[ \mu_q - \gamma \lambda_t \sigma_q^2 - (1 - \gamma) \varphi_{P,q} \sigma_P \sigma_q = 0 \quad \text{(A.8)} \]

Solving for \( \lambda_t \) in Equation (A.7) gives Equation (12) in the text. Furthermore, using Equation (A.6) and (A.7) one can find the constant \( A \) and the constants \( \eta_j \), yielding

\[ A = \prod_{j=1}^{N} a_j^{a_j} \left\{ \varepsilon \delta + (1 - \varepsilon) r_{CE} \right\}^{\frac{1}{1-\varepsilon}} \quad \text{(A.9)} \]
\[ \eta_j = a_j (\varepsilon \delta + (1 - \varepsilon) r^{CE}) \]  

(A.10)

where, as defined in the text, \( r^{CE} = \bar{r} - \gamma (\bar{\sigma}^2 / 2) \), with \( \bar{r} = r + \left( \mu - \varphi_{p,q} \sigma_p \sigma_q \right) \lambda^* - \Pi + \bar{\sigma}^2 \).

and \( \bar{\sigma}^2 = \sigma_q^2 (\lambda^*)^2 + \sigma_p^2 - 2 \lambda^* \varphi_{p,q} \sigma_p \sigma_q \).

The demand for good \( j \) is then

\[ x_{j,t} = a_j (\varepsilon \delta + (1 - \varepsilon) r^{CE}) \left( w_t / p_{j,t} \right). \]  

(A.11)

Using Equation (A.11) one finds that total expenditures equal

\[ C_t = \sum_{i=1}^{N} x_{i,t} p_i = (\varepsilon \delta + (1 - \varepsilon) r^{CE}) w_t. \]  

(A.12)

**Proof of Proposition 1.**

When \( \varphi_{j,t} = 1 \) and \( \sigma_j = \sigma_i \) for all \( j \) and \( i \) then \( \sigma_p^2 = \sum_{j=1}^{N} a_j \frac{\sigma_j^2}{2} \) and \( \Pi = \pi \). Using this and also setting \( \lambda^* = 0 \) in the derivation of Proposition 2 we obtain Proposition 1 in the text.

**Proof of Proposition 3.**

Plugging A.9 back into A.2 we obtain

\[ u(w_t, p_t) = \left( \prod_{j=1}^{N} a_j^{(1 - \gamma)} \right) \left( \varepsilon \delta + (1 - \varepsilon) r^{CE} \right)^{1 - \gamma} w_t^{(1 - \gamma)} (1 - \gamma)^{-1} \]  

(A.18)

We defined the welfare cost of volatility as

\[ u \left( \frac{w_0}{p_0}, r^{CE} \right) = u \left( (1 - k) \frac{w_0}{p_0}, \bar{r}^{CE} \right), \]

where \( \bar{r}^{CE} = r - \pi + \frac{\mu^2}{\gamma \sigma_q^2} \) is the certainty equivalent real interest rate when all prices grow at a constant rate \( \pi \). Using A.18 and solving for \( k \) we obtain the expression in Proposition 3.
References


Table 1. Parameter estimates and calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu = \omega_q - r )</td>
<td>0.0541</td>
</tr>
<tr>
<td>( \sigma_q )</td>
<td>0.1649</td>
</tr>
<tr>
<td>( \sigma_p = \left( \frac{1}{2} \sum_{j=1}^{N} \sum_{i=1}^{N} a_j \phi_{j,i} \sigma_j \sigma_i \right)^{1/2} )</td>
<td>0.0233</td>
</tr>
<tr>
<td>( \left( \sum_{j=1}^{N} a_j \frac{\sigma_j^2}{2} \right)^{1/2} )</td>
<td>0.0344</td>
</tr>
<tr>
<td>( \phi_{p,q} = \frac{\sum_{j=1}^{N} a_j \phi_{j,q} \sigma_j}{\sigma_p} )</td>
<td>-0.1952</td>
</tr>
</tbody>
</table>

Table 2. Strength of Precautionary saving motive \[1 - C_t(r^CE)/C_t(r^{CE})\]

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>1</th>
<th>2</th>
<th>3.27</th>
<th>6</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>-0.55%</td>
<td>-0.61%</td>
<td>0.00%</td>
<td>3.760%</td>
<td>14.89%</td>
<td>67.93%</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.55%</td>
<td>-0.60%</td>
<td>0.00%</td>
<td>3.542%</td>
<td>13.55%</td>
<td>57.48%</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.47%</td>
<td>-0.45%</td>
<td>0.00%</td>
<td>1.882%</td>
<td>5.72%</td>
<td>17.22%</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.21%</td>
<td>-0.14%</td>
<td>0.00%</td>
<td>0.361%</td>
<td>0.92%</td>
<td>2.36%</td>
</tr>
</tbody>
</table>

Note: A negative sign indicates that consumption is higher with uncertainty.

Table 3. Welfare cost of price uncertainty (\( k \))

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>1</th>
<th>2</th>
<th>3.27</th>
<th>6</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>-0.58%</td>
<td>-0.64%</td>
<td>0.00%</td>
<td>3.95%</td>
<td>15.61%</td>
<td>69.79%</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.61%</td>
<td>-0.66%</td>
<td>0.00%</td>
<td>3.93%</td>
<td>14.93%</td>
<td>61.33%</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.94%</td>
<td>-0.91%</td>
<td>0.00%</td>
<td>3.73%</td>
<td>11.11%</td>
<td>31.47%</td>
</tr>
<tr>
<td>0.9</td>
<td>-2.11%</td>
<td>-1.44%</td>
<td>0.00%</td>
<td>3.55%</td>
<td>8.85%</td>
<td>21.23%</td>
</tr>
</tbody>
</table>

Note: A negative sign indicates that welfare is higher with uncertainty.